

Clustered Defaults

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Why consider clustered defaults?

- The recent financial crisis exposed serious problems in credit portfolio modeling. It also raised questions about our understanding of the vulnerability of the corporate sector.
- Credit rating models/practices are also being challenged.
- Defaults in a credit portfolio of many obligors or in an economy populated with firms tend to occur in waves. This may simply reflect their sharing of common risk factors and/or manifest their systemic linkages via credit chains.

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Top-down vs. bottom-up approaches

- The top-down approach directly models the aggregate behavior at the portfolio level, and is intended for answering questions only concerning the overall portfolio. Examples are Arnsdorf and Halperin (2007), Cont and Minca (2007), Giesecke and Kim (2007), and Longstaff and Rajan (2007).
- The bottom-up approach models individual names together by specifying their joint behavior. Such models are in principle more useful, but may deliver unsatisfactory performance at the aggregate level due to the built-in constraints from modeling individual obligors. Examples are Li (2000), Shumway (2001), Andersen, et al (2003), Duffie, et al (2007), Duffie, et al (2008), and Peng and Kou (2009).

The doubly stochastic intensity model

- N_{it} is a Poisson process with stochastic intensity λ_{it} , which is a function of stochastic covariates. There is a directional relationship between λ_{it} and N_{it} ; that is, λ_{it} affects N_{it} but not the other way around.
- N_{it} are independent across obligors after conditioning on stochastic covariates.
- Default occurs once $N_{it} = 1$.

Note that the joint default rate always equals zero.

Evidence against the standard intensity model

- Azizpour and Giesecke (2008) reported 24 rail firm defaults in a single day on June 21, 1970.
- Das, *et al* (2007) showed by a battery of tests that the standard intensity model such as Duffie, *et al* (2007) simply does not generate enough default clustering as in the observed data.

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Alternative approaches

- In the top-down context, Azizpour and Giesecke (2008) employed a self-exciting model.
- Duffie, *et al* (2008) built "frailty" into the Poisson intensity model by introducing latent variables so as to increase default clustering.
- Peng and Kou (2009) made the cumulative intensity process to have jumps, instead of jumps in the local intensity process. Their approach amounts to making the local intensity a Dirac delta function, and thus the joint default intensity does not vanish locally.

The hierarchical intensity model

For obligor (i, j), which is the *j*-th member of the *i*-the group where $i = 1, \dots, K$ and $j = 1, \dots, n_i$,

$$dM_{ijt} = \chi_{ijt}dN_{ct} + \zeta_{ijt}dN_{it} + dN_{ijt}.$$

where $N_{c0} = N_{i0} = N_{ij0} = 0$, and χ_{ijt} (or ζ_{ijt}) is a Bernoulli random variable taking value of 1 with a probability of p_{ijt} (or q_{ijt}) and 0 with a probability of $1 - p_{ijt}$ (or $1 - q_{ijt}$). χ_{ijt} and ζ_{ijt} are independent of each other and also independent across different obligors.

Note that the joint default rate is no longer zero.

Reducing to the standard intensity model

The default component of the hierarchical intensity model viewed individually can be reduced to

$$dM_{ijt} \stackrel{d}{=} \chi^*_{ijt} dN^*_{ijt}$$

where $\stackrel{d}{=}$ stands for distributional equivalence; N_{ijt}^* is a Poisson process with the intensity equal to $\lambda_{ct} + \lambda_{it} + \lambda_{ijt}$; and χ_{ijt}^* is a Bernoulli random variable taking value of 1 with a probability of p_{ijt}^* and 0 with a probability of $1 - p_{ijt}^*$. Note that

$$p_{ijt}^* = \frac{\lambda_{ct}}{\lambda_{ct} + \lambda_{it} + \lambda_{ijt}} p_{ijt} + \frac{\lambda_{it}}{\lambda_{ct} + \lambda_{it} + \lambda_{ijt}} q_{ijt} + \frac{\lambda_{ijt}}{\lambda_{ct} + \lambda_{it} + \lambda_{ijt}}$$

It is clear that $\chi_{ijt}^* dN_{ijt}^*$ is also equivalent in distribution to a Poisson process with intensity of $p_{ijt}^*(\lambda_{ct} + \lambda_{it} + \lambda_{ijt})$ with respect to the default time.

If we look at an obligor individually, the hierarchical intensity model is equivalent to the Duffie, *et al* (2007) model. But putting two or more obligors together, the two models are not equivalent, i.e., $(M_{ijt}, M_{klt}) \stackrel{d}{\neq} (\int_{0}^{t} \chi_{ijs}^{*} dN_{ijs}^{*}, \int_{0}^{t} \chi_{kls}^{*} dN_{kls}^{*}).$

Computing predicted default frequencies

The DSW model:

$$\begin{aligned} &Prob^{DSW}(U=0) \\ &= \prod_{i=1}^{K} \prod_{j=1}^{n_i} e^{-\lambda_{ijt}\Delta t} \\ &Prob^{DSW}(U=1) \\ &= \sum_{i=1}^{K} \sum_{j=1}^{n_i} \left((1 - e^{-\lambda_{ijt}\Delta t}) \prod_{m=1}^{K} \prod_{l=1}^{n_i} e^{-\lambda_{mlt}\Delta t \mathbb{1}_{\{(m,l)\neq(i,j)\}}} \right) \\ &\vdots \end{aligned}$$

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Computing predicted default frequencies

A restricted HIM:

$$\begin{aligned} &Prob^{rHIM}(U=0) \\ &= e^{-\lambda_{ct}\Delta t} + (1 - e^{-\lambda_{ct}\Delta t}) \prod_{i=1}^{K} \prod_{j=1}^{n_{i}} (1 - p_{ijt}) \\ &Prob^{rHIM}(U=1) \\ &= (1 - e^{-\lambda_{ct}\Delta t}) \sum_{i=1}^{K} \sum_{j=1}^{n_{i}} \left(p_{ijt} \prod_{m=1}^{K} \prod_{l=1}^{n_{i}} (1 - p_{mlt})^{1_{\{(m,l)\neq(i,j)\}}} \right) \\ &\vdots \end{aligned}$$

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Default correlation and double default probability

Let $\tau_{ij} = \inf(t; M_{ijt} \ge 1), \ \tau_{ij}^* = \inf(t; \int_0^t \chi_{ijs}^* dN_{ijs}^* \ge 1)$ where N_{ijs}^* are independent Poisson processes mimicking the Duffie, *et al* (2007) model. In addition, the non-default exit time is $\phi_{ij} = \inf(t; L_{ijt} \ge 1).$

Double default probability:

$$E_0\left(1_{\{\tau_{ij} \le t \land \phi_{ij}\}} 1_{\{\tau_{kl} \le t \land \phi_{kl}\}}\right) \ge E_0\left(1_{\{\tau_{ij}^* \le t \land \phi_{ij}\}} 1_{\{\tau_{kl}^* \le t \land \phi_{kl}\}}\right)$$

Default correlation:

$$Corr_0\left(1_{\{\tau_{ij}\leq t\wedge\phi_{ij}\}},1_{\{\tau_{kl}\leq t\wedge\phi_{kl}\}}\right)\geq Corr_0\left(1_{\{\tau_{ij}^*\leq t\wedge\phi_{ij}\}},1_{\{\tau_{kl}^*\leq t\wedge\phi_{kl}\}}\right)$$

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Decomposable likelihood function

$$\begin{aligned} \mathcal{L}(\theta,\varphi;D_T,I_T,V) &= \mathcal{L}(\varphi;D_T,I_T,V) + \mathcal{L}(\theta;D_T) \\ &= \sum_{t=2}^T \ln\left(A_t(\varphi;D_t,I_t,V)\right) + \mathcal{L}(\theta;D_T) \end{aligned}$$

where

$$\begin{array}{l} A_{t}(\varphi; D_{t}, I_{t}, V) \\ = & \left(\prod_{i=1}^{K} \prod_{j=1}^{n_{i}} B_{ijt}\right) \left\{ e^{-\lambda_{c}(t-1)\Delta t} \prod_{i=1}^{K} \left(e^{-\lambda_{i}(t-1)\Delta t} \prod_{j=1}^{n_{i}} C_{ijt}^{(1)} + (1-e^{-\lambda_{i}(t-1)\Delta t}) \prod_{j=1}^{n_{i}} C_{ijt}^{(2)} \right) \\ & + (1-e^{-\lambda_{c}(t-1)\Delta t}) \prod_{i=1}^{K} \left(e^{-\lambda_{i}(t-1)\Delta t} \prod_{j=1}^{n_{i}} C_{ijt}^{(3)} + (1-e^{-\lambda_{i}(t-1)\Delta t}) \prod_{j=1}^{n_{i}} C_{ijt}^{(4)} \right) \right\}$$

Note that we are able to separate the parameters governing defaults from those determining other forms of exits. (z) = -2

Data

- The US firms over the period of Jan 1991 to Dec 2008 with a monthly frequency.
- Data sources: the accounting data (lagged 3 months) from the Compustat quarterly and annual database; the stock market data are from the CRSP monthly file; the de-listing/bankruptcy data are from the CRSP file which is then coupled with with the default data from the Bloomberg CACS function; and the interest rates are from the US Federal Reserve.
- There are 872 bankruptcies/defaults. The sample consists of 14,401 firms with 1,317,217 firm-month observations. Among them, 2,844 companies are financial (SIC code between 6000 and 6999) with 249,382 firm-month observations.

The variables used in the models

- Follow Duffie, *et al* (2007) to use the four covariates: trailing one-year S&P 500 index return, three-month Treasury bill rate, firm's trailing one-year return, and firm's distance-to-default.
- The KMV assumption sets the firm's debt maturity to one year and size to the sum of the short-term debt and 50% of the long-term debt. This KMV assumption is typically used only for non-financial firms. We include financial firms by adding to the KMV debt a fraction of other liabilities. Individual firm's fraction is estimated by applying the maximum likelihood estimation method of Duan (1994).

Estimation results: Jan 1991 to Dec 2008, monthly

		DSW	HIM		
Common Shock Intensity Function	Intercept		17.7945** (8.6799)	5.4012*** (1.6272)	16.6699** (8.0790)
	Average DTD all		-3.8191* (2.0597)		
	Average DTD financial			-0.6188** (0.2766)	
	Average DTD nonfinancial			All services	-3.9382* (2.1505)
p Function	Intercept	8 8	-6.324.5*** (0.2049)	-7.6097*** (0.1988)	-6.2811*** (0.2006)
	DTD		0.949*** (0.2371)	-0.4466*** (0.0473)	0.9382**** (0.2352)
Firm-specific Shock Intensity Function	Intercept	-4.9942*** (0.0794)	-0.0381 (0.0236)	-6.3434*** (0.2072)	-0.0393* (0.0235)
	Trailing 1-Year S P500 Return	0.7727*** (0.2038)	-0.8087*** (0.0390)	0.9324*** (0.2380)	-0.8099*** (0.0386)
	3-Month Treasury Rate	-0.0505** (0.0199)	-4.542*** (0.2432)	-0.0375 (0.0235)	-4.4967*** (0.2389)
	DTD	-0.781 <i>5</i> *** (0.0254)	-7 <i>96</i> 97*** (0.1676)	-0.8074*** (0.0388)	-8.0156*** (0.1780)
	Trailing 1-Year Return	-3.0017*** (0.0824)	-0.4389*** (0.0452)	-4.5659*** (0.2515)	-0.4318*** (0.0461)
Log-likelihood		-5359.9	-5239.9	-5240.0	-5242.1

Default frequency: entire sample



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Default frequency: bottom 3rd financial firm DTD



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Default frequency: top 3rd financial firm DTD



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GAP comparison



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KL distance and DTD over the sample period



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