

Challenges of the Emissions Markets

(Pricing Options on CO₂)

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- **Cap & Trade Schemes for CO₂ Emissions**

- Kyoto Protocol
- Mandatory Carbon Markets **(EU ETS, RGGI since 01/01/09)**
- Lessons learned from the EU Experience

- **Mathematical (Equilibrium) Models**

- Price Formation for Goods and Emission Allowances
- New Designs and Alternative Schemes
- Calibration & Option Pricing

- **Computer Implementations**

- Several case studies (Texas, Japan)
- Practical Tools for Regulators and Policy Makers

- **No (Significant) Emissions Reduction**

- DID Emissions go down?
- Yes, but as part of an existing trend

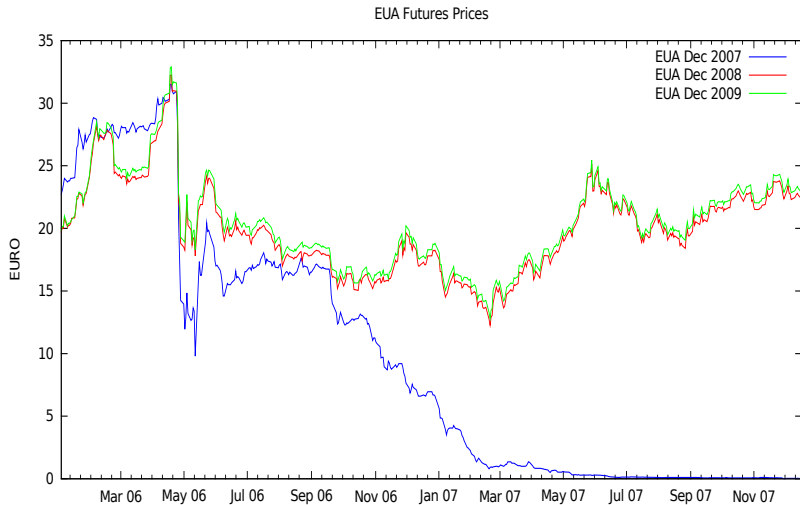
- **Significant Increase in Prices**

- Cost of Pollution passed along to the "end-consumer"
- Small proportion (40%) of polluters involved in EU ETS

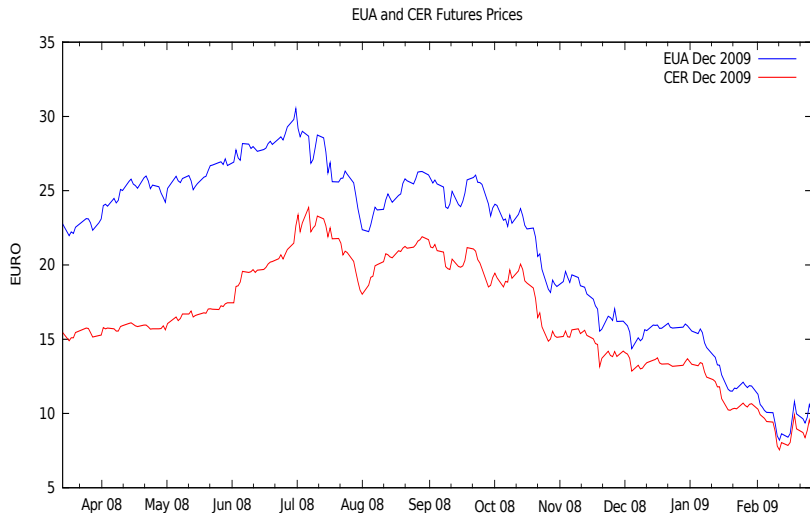
- **Windfall Profits**

- Cannot be avoided
- Proposed Remedies
 - Stop Giving Allowance Certificates Away for Free !
 - Auctioning

Falling Carbon Prices: What Happened?



CDM: Can we Explain CER Prices?



Description of the Economy

- **Finite set $\mathcal{I}$ of risk neutral firms**
- **Producing a finite set $\mathcal{K}$ of goods**
- Firm $i \in \mathcal{I}$ can use **technology** $j \in \mathcal{J}^{i,k}$ to produce good $k \in \mathcal{K}$
- **Discrete time** $\{0, 1, \dots, T\}$
- **No Discounting** Work with T -Forward Prices
- **Inelastic Demand**

$$\{D^k(t); t = 0, 1, \dots, T - 1, k \in \mathcal{K}\}.$$

●

At inception of program (i.e. time $t = 0$)

- **INITIAL DISTRIBUTION** of **allowance certificates**

$$\theta_0^i \quad \text{to firm } i \in \mathcal{I}$$

- Set **PENALTY** π for emission unit **NOT** offset by allowance certificate at end of **compliance period**

Extensions (not discussed in this talk)

- **Risk aversion** and agent preferences (existence theory easy)
- **Elastic** demand (e.g. smart meters for electricity)
- **Multi-period models** with lending, borrowing and withdrawal (more realistic)
-

Goal of Equilibrium Analysis

Find **two stochastic processes**

- **Price of one allowance**

$$A = \{A_t\}_{t \geq 0}$$

- **Prices of goods**

$$S = \{S_t^k\}_{k \in K, t \geq 0}$$

satisfying the usual conditions for the existence of a

competitive equilibrium

(to be spelled out below).

Individual Firm Problem

During each time period $[t, t + 1)$

- Firm $i \in \mathcal{I}$ **produces** $\xi_t^{i,j,k}$ of good $k \in \mathcal{K}$ with technology $j \in \mathcal{J}^{i,k}$
- Firm $i \in \mathcal{I}$ **holds** a position θ_t^i in emission credits

$$\begin{aligned} L^{A,S,i}(\theta^i, \xi^i) := & \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (s_t^k - c_t^{i,j,k}) \xi_t^{i,j,k} \\ & + \theta_0^i A_0 + \sum_{t=0}^{T-1} \theta_{t+1}^i (A_{t+1} - A_t) - \theta_{T+1}^i A_T \\ & - \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_{T+1}^i)^+ \end{aligned}$$

where

$$\Gamma^i \text{ random,} \quad \Pi^i(\xi^i) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} e^{i,j,k} \xi_t^{i,j,k}$$

Problem for (risk neutral) firm $i \in \mathcal{I}$

$$\max_{(\theta^i, \xi^i)} \mathbb{E}\{L^{A,S,i}(\theta^i, \xi^i)\}$$

Equilibrium Definition for Emissions Market

The processes $A^* = \{A_t^*\}_{t=0,1,\dots,T}$ and $S^* = \{S_t^*\}_{t=0,1,\dots,T}$ form an equilibrium if for each agent $i \in \mathcal{I}$ there exist strategies $\theta^{*i} = \{\theta_t^{*i}\}_{t=0,1,\dots,T}$ (**trading**) and $\xi^{*i} = \{\xi_t^{*i}\}_{t=0,1,\dots,T}$ (**production**)

- **(i) All financial positions are in constant net supply**

$$\sum_{i \in I} \theta_t^{*i} = \sum_{i \in I} \theta_0^i, \quad \forall t = 0, \dots, T + 1$$

- **(ii) Supply meets Demand**

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}^{i,k}} \xi_t^{*i,j,k} = D_t^k, \quad \forall k \in \mathcal{K}, t = 0, \dots, T - 1$$

- **(iii) Each agent $i \in I$ is satisfied by its own strategy**

$$\mathbb{E}[L^{A^*, S^*, i}(\theta^{*i}, \xi^{*i})] \geq \mathbb{E}[L^{A^*, S^*, i}(\theta^i, \xi^i)] \quad \text{for all } (\theta^i, \xi^i)$$

Business As Usual (i.e. $\pi = 0$)

The corresponding prices of the goods are

$$S_t^{*k} = \max_{i \in \mathcal{I}, j \in \mathcal{J}^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}},$$

Classical **MERIT ORDER**

- At each time t and for each good k
- Production technologies ranked by increasing production costs $C_t^{i,j,k}$
- Demand D_t^k met by producing from the cheapest technology first
- Equilibrium spot price is the marginal cost of production of the most expansive production technology used to meet demand

Business As Usual

(typical scenario in Deregulated **electricity markets**)

Necessary Conditions

Assume

- (A^*, S^*) is an equilibrium
- (θ^{*i}, ξ^{*i}) optimal strategy of agent $i \in I$

then

- The allowance price A^* is a **bounded martingale** in $[0, \pi]$
- Its terminal value is given by

$$A_T^* = \pi \mathbf{1}_{\{\Gamma^i + \Pi(\xi^{*i}) - \theta_{T+1}^{*i} \geq 0\}} = \pi \mathbf{1}_{\{\sum_{i \in \mathcal{I}} (\Gamma^i + \Pi(\xi^{*i}) - \theta_0^{*i}) \geq 0\}}$$

- The **spot prices** S^{*k} of the goods and the **optimal production strategies** ξ^{*i} are given by the **merit order** for the equilibrium with **adjusted costs**

$$\tilde{C}_t^{i,j,k} = C_t^{i,j,k} + e^{i,j,k} A_t^*$$

Existence by Social Cost Minimization

- Overall production costs

$$C(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} \xi_t^{i,j,k} C_t^{i,j,k}.$$

- Overall cumulative emissions

$$\Gamma := \sum_{i \in I} \Gamma^i \quad \Pi(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)} e^{i,j,k} \xi_t^{i,j,k},$$

- Total allowances

$$\theta_0 := \sum_{i \in I} \theta_0^i$$

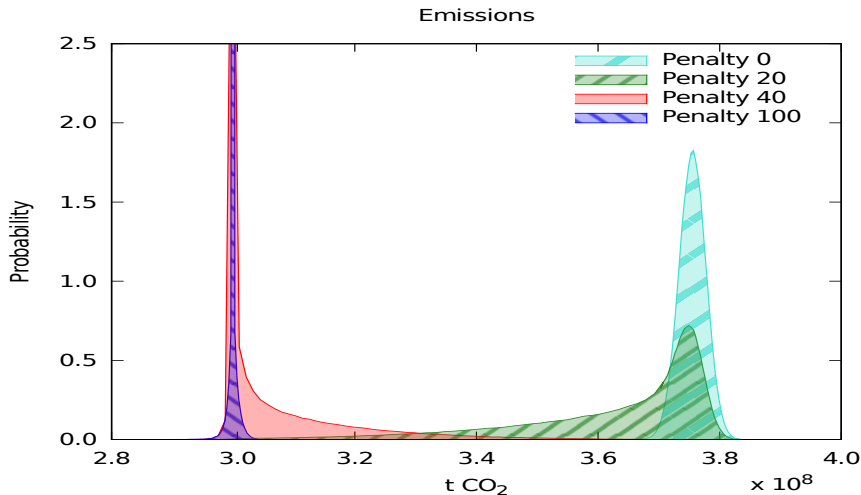
The **total social costs from production and penalty payments**

$$G(\xi) := C(\xi) + \pi(\Gamma + \Pi(\xi) - \theta_0)^+$$

We introduce the global optimization problem

$$\xi^* = \arg \inf_{\xi \text{ meets demands}} \mathbb{E}[G(\xi)],$$

Effect of the Penalty on Emissions



Costs in a Cap-and-Trade

- Consumer Burden

$$SC = \sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k.$$

- Reduction Costs (producers' burden)

$$\sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k}$$

- Excess Profit

$$\sum_t \sum_k (S_t^{k,*} - S_t^{k,BAU*}) D_t^k - \sum_t \sum_{i,j,k} (\xi_t^{i,j,k*} - \xi_t^{BAU,i,j,k*}) C_t^{i,j,k} - \pi \left(\sum_t \sum_{ijk} \xi_t^{ijk} e_t^{ijk} - \theta_0 \right)^+$$

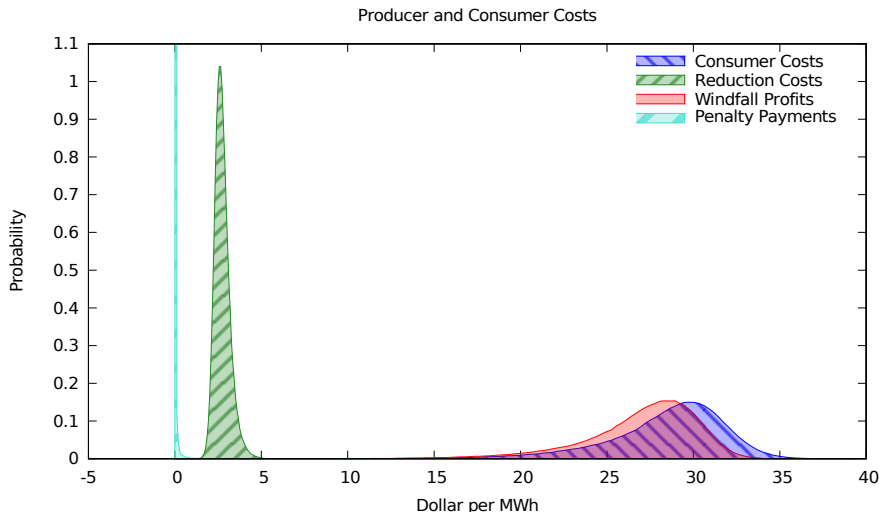
- Windfall Profits

$$WP = \sum_{t=0}^{T-1} \sum_{k \in K} (S_t^{*k} - \hat{S}_t^k) D_t^k$$

where

$$\hat{S}_t^k := \max_{i \in I, j \in J^{i,k}} C_t^{i,j,k} \mathbf{1}_{\{\xi_t^{*i,j,k} > 0\}}.$$

Costs in a Cap-and-Trade Scheme



Histograms of consumer costs, social costs, windfall profits and penalty payments of a standard cap-and-trade scheme calibrated to reach the emissions target with 95% probability and BAU.

One of many Possible Generalizations

Introduction of **Taxes / Subsidies**

$$\begin{aligned}\ddot{L}^{A,S,i}(\theta^i, \xi^i) = & - \sum_{t=0}^{T-1} G_t^i + \sum_{k \in K} \sum_{j \in J^{i,k}} \sum_{t=0}^{T-1} (S_t^k - C_t^{i,j,k} - H_t^k) \xi_t^{i,j,k} \\ & + \sum_{t=0}^{T-1} \theta_t^i (A_{t+1} - A_t) - \theta_T^i A_T \\ & - \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_T^i)^+.\end{aligned}$$

In this case

- In equilibrium, **production** and **trading** strategies remain the same $(\theta^\dagger, \xi^\dagger) = (\theta^*, \xi^*)$
- **Abatement costs** and **Emissions reductions** are also the same
- New equilibrium prices $(A^\dagger, S^\dagger)$ given by

$$A_t^\dagger = A_t^* \quad \text{for all } t = 0, \dots, T \quad (1)$$

$$S_t^{\dagger k} = S_t^{*k} + H_t^k \quad \text{for all } k \in K, t = 0, \dots, T-1 \quad (2)$$

- Cost of the tax passed along to the end consumer

- **Currently Regulator Specifies**

- Penalty π
- Overall Certificate Allocation $\theta_0 (= \sum_{i \in I} \theta_0^i)$

- **Alternative Scheme (Still) Controlled by Regulator**

- (i) **Sets penalty level** π

- (ii) **Allocates allowances**

- θ_0' at inception of program $t = 0$
- then **proportionally to production**

$y \xi_t^{i,j,k}$ to agent i for producing $\xi_t^{i,j,k}$ of good k with technology j

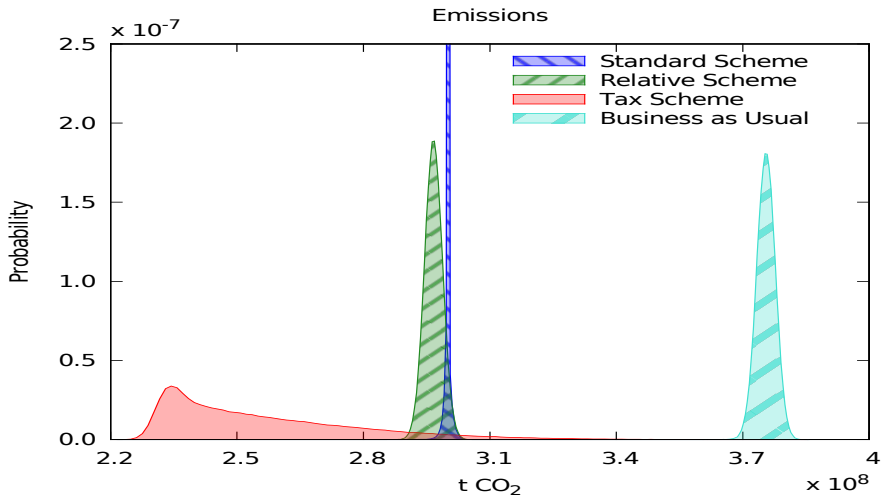
- (iii) **Calibrates** y , e.g. in **expectation**.

$$y = \frac{\theta_0 - \theta_0'}{\sum_{t=0}^{T-1} \sum_{k \in K} \mathbb{E}\{D_t^k\}}$$

So total number of credit allowance is the same in expectation, i.e.

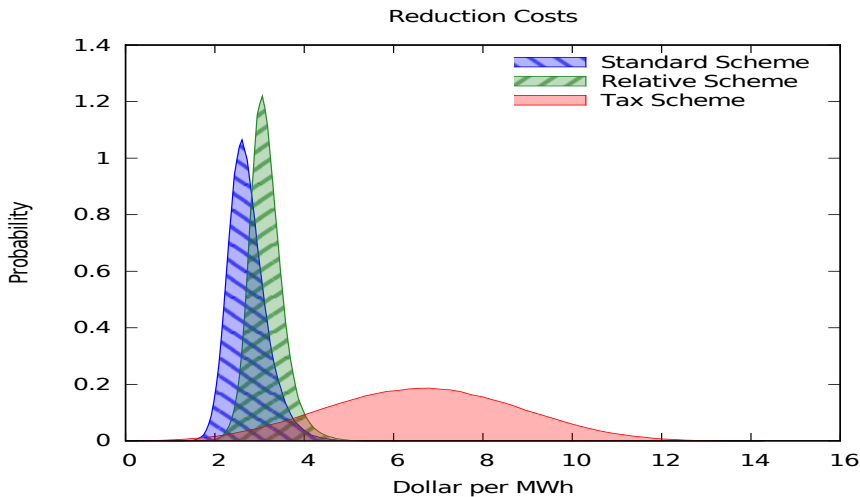
$$\theta_0 = \mathbb{E}\{\theta_0' + y \sum_{t=0}^{T-1} \sum_{k \in K} D_t^k\}$$

Yearly Emissions Equilibrium Distributions



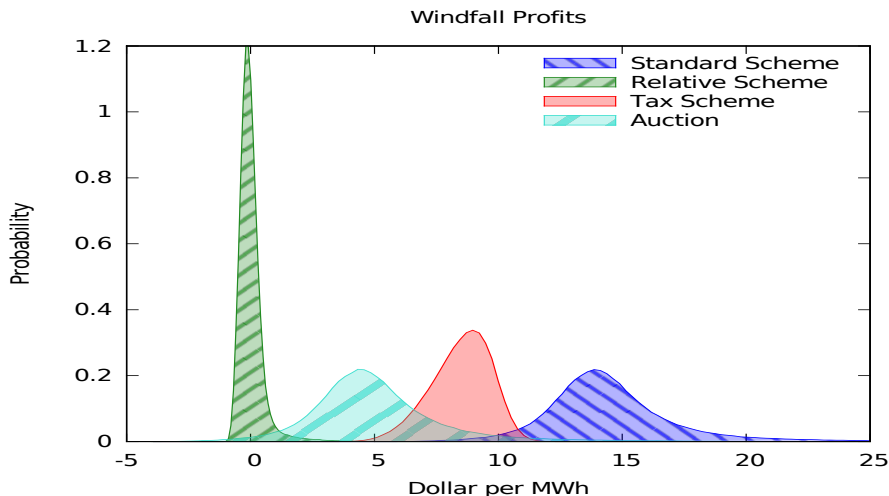
Yearly emissions from electricity production for the Standard Scheme, the Relative Scheme, a Tax Scheme and BAU.

Abatement Costs



Yearly abatement costs for the Standard Scheme, the Relative Scheme and a Tax Scheme.

Windfall Profits



Histograms of the yearly distribution of windfall profits for the Standard Scheme, a Relative Scheme, a Standard Scheme with 100% Auction and a Tax Scheme

(Uhrig-Homburg-Wagner, R.C - Hinz)

- Emissions Cap-and-Trade Markets **SOON** to exist in the US (and Canada, Australia, Japan,)
- Liquid **Option** Market **ALREADY** exists in Europe
 - Underlying $\{A_t\}_t$ non-negative martingale with **binary terminal value**
 - Think of A_t as of a binary option
 - Underlying of binary option should be *Emissions*
- Need for **Formulae** (closed or computable)
 - Prices and Hedges difficult to compute (only numerically)
 - to study effect of announcements (**Cetin et al.**)
- **Reduced Form Models**

Option quotes on Jan. 3, 2008

Option Maturity	Option Type	Volume	Strike	Allowance Price	Implied Vol	Settlement Price
Dec-08	Call	150,000	24.00	23.54	50.50%	4.19
Dec-08	Call	500,000	26.00	23.54	50.50%	3.50
Dec-08	Call	25,000	27.00	23.54	50.50%	3.20
Dec-08	Call	300,000	35.00	23.54	50.50%	1.56
Dec-08	Call	1,000,000	40.00	23.54	50.50%	1.00
Dec-08	Put	200,000	15.00	23.54	50.50%	0.83

Option quotes on Jan. 4, 2008

Option Maturity	Option Type	Volume	Strike	Allowance Price	Implied Vol	Settlement Price
Dec-08	Cal	200,000	22.00	23.55	51.25%	5.06
Dec-08	Call	150,000	26.00	23.55	51.25%	3.57
Dec-08	Call	450,000	27.00	23.55	51.25%	3.27
Dec-08	Call	100,000	28.00	23.55	51.25%	2.99
Dec-08	Call	125,000	29.00	23.55	51.25%	2.74
Dec-08	Call	525,000	30.00	23.55	51.25%	2.51
Dec-08	Call	250,000	40.00	23.55	51.25%	1.04
Dec-08	Call	700,000	50.00	23.55	51.25%	0.45
Dec-08	Put	1,000,000	14.00	23.55	51.25%	0.64
Dec-08	Put	200,000	15.00	23.55	51.25%	0.86
Dec-08	Put	200,000	15.00	23.55	51.25%	0.86
Dec-08	Put	400,000	16.00	23.55	51.25%	1.13
Dec-08	Put	100,000	17.00	23.55	51.25%	1.43
Dec-08	Put	1,000,000	18.00	23.55	51.25%	1.78
Dec-08	Put	500,000	20.00	23.55	51.25%	2.60
Dec-08	Put	200,000	21.00	23.55	51.25%	3.07
Dec-08	Put	200,000	22.00	23.55	51.25%	3.57

Reduced Form Models and Calibration

Allowance price should be of the form

$$A_t = \pi \mathbb{E}\{\mathbf{1}_N \mid \mathcal{F}_t\}$$

for a non-compliance set $N \in \mathcal{F}_t$. Choose

$$N = \{\Gamma_T \geq 1\}$$

for a random variable Γ_T representing the normalized emissions at compliance time. So

$$A_t = \pi \mathbb{E}\{\mathbf{1}_{\{\Gamma_T \geq 1\}} \mid \mathcal{F}_t\}, \quad t \in [0, T]$$

We choose Γ_T in a parametric family

$$\Gamma_T = \Gamma_0 \exp \left[\int_0^T \sigma_s dW_s - \frac{1}{2} \int_0^T \sigma_s^2 ds \right]$$

for some square integrable deterministic function

$$(0, T) \ni t \mapsto \sigma_t$$

Dynamic Price Model for $a_t = \frac{1}{\pi} A_t$

- a_t is given by

$$a_t = \Phi \left(\frac{\Phi^{-1}(a_0) \sqrt{\int_0^T \sigma_s^2 ds} + \int_0^t \sigma_s dW_s}{\sqrt{\int_t^T \sigma_s^2 ds}} \right) \quad t \in [0, T]$$

where Φ is standard normal c.d.f.

- a_t solves the SDE

$$da_t = \Phi'(\Phi^{-1}(a_t)) \sqrt{z_t} dW_t$$

where the positive-valued function $(0, T) \ni t \mapsto z_t$ is given by

$$z_t = \frac{\sigma_t^2}{\int_t^T \sigma_u^2 du}, \quad t \in (0, T)$$

Risk Neutral Densities

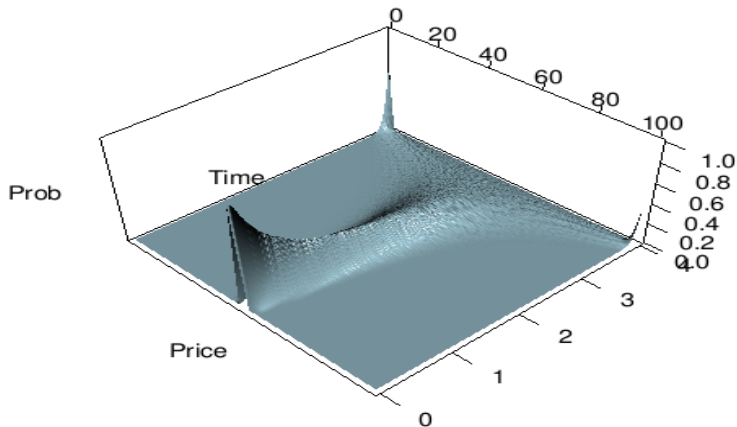


Figure: Histograms for each day of a 4 yr compliance period of 10^5 simulated risk neutral allowance price paths.

Aside: Binary Martingales as Underliers

Allowance prices are given by $A_t = \pi a_t$ where $\{a_t\}_{0 \leq t \leq T}$ satisfies

- $\{a_t\}_t$ is a martingale
- $0 \leq a_t \leq 1$
- $\mathbb{P}\{\lim_{t \rightarrow T} a_t = 1\} = 1 - \mathbb{P}\{\lim_{t \rightarrow T} a_t = 0\} = p$ for some $p \in (0, 1)$

The model

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t}dW_t$$

suggests looking for martingales $\{Y_t\}_{0 \leq t < \infty}$ satisfying

- $0 \leq Y_t \leq 1$
- $\mathbb{P}\{\lim_{t \rightarrow \infty} Y_t = 1\} = 1 - \mathbb{P}\{\lim_{t \rightarrow \infty} Y_t = 0\} = p$ for some $p \in (0, 1)$

and do a **time change** to get back to the (compliance) interval $[0, T]$

Feller's Theory of 1-D Diffusions

Gives conditions for the SDE

$$da_t = \Theta(a_t)dW_t$$

for $x \mapsto \Theta(x)$ satisfying

- $\Theta(x) > 0$ for $0 < x < 1$
- $\Theta(0) = \Theta(1) = 0$

to

- Converge to the boundaries 0 and 1
- NOT explode (i.e. NOT reach the boundaries in finite time)

Interestingly enough the solution of

$$dY_t = \Phi'(\Phi^{-1}(Y_t))dW_t$$

IS ONE OF THEM !

More Explicit Examples

The SDE

$$dX_t = \sqrt{2}dW_t + X_t dt$$

has the solution

$$X_t = e^t \left(x_0 + \int_0^t e^{-s} dW_s \right)$$

and

$$\begin{aligned} \lim_{t \rightarrow \infty} X_t = +\infty & \quad \text{on the set } \left\{ \int_0^\infty e^{-s} dW_s > -x_0 \right\} \\ \lim_{t \rightarrow \infty} X_t = -\infty & \quad \text{on the set } \left\{ \int_0^\infty e^{-s} dW_s < -x_0 \right\} \end{aligned}$$

Moreover Φ is **harmonic** so if we choose

$$Y_t = \Phi(X_t)$$

we have a martingale with the desired properties.

Other (explicit) examples can be constructed from **Ph. Carmona - Petit - Yor** on Dufresne formula (jump processes by **M. Tehranchi**)

Has to Be Historical !!!!

- Choose **Constant** Market Price of Risk
- **Two-parameter** Family for Time-change

$$\{z_t(\alpha, \beta) = \beta(T - t)^{-\alpha}\}_{t \in [0, T]}, \quad \beta > 0, \alpha \geq 1.$$

Volatility function $\{\sigma_t(\alpha, \beta)\}_{t \in (0, T)}$ given by

$$\begin{aligned} \sigma_t(\alpha, \beta)^2 &= z_t(\alpha, \beta) e^{-\int_0^t z_u(\alpha, \beta) du} \\ &= \begin{cases} \beta(T - t)^{-\alpha} e^{\beta \frac{T^{-\alpha+1} - (T-t)^{-\alpha+1}}{-\alpha+1}} & \text{for } \beta > 0, \alpha > 1 \\ \beta(T - t)^{\beta-1} T^{-\beta} & \text{for } \beta > 0, \alpha = 1 \end{cases} \end{aligned}$$

Maximum Likelihood

Sample Data

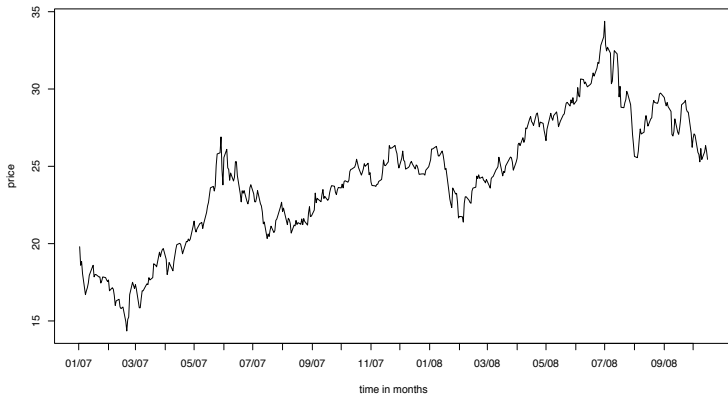


Figure: Future prices on EUA with maturity Dec. 2012

Call Option Price in One Period Model

for $\alpha = 1$, $\beta > 0$, the price of an European call with strike price $K \geq 0$ written on a one-period allowance futures price at time $\tau \in [0, T]$ is given at time $t \in [0, \tau]$ by

$$\begin{aligned} C_t &= e^{-\int_t^\tau r_s ds} \mathbb{E}\{(A_\tau - K)^+ \mid \mathcal{F}_t\} \\ &= \int (\pi \Phi(x) - K)^+ N(\mu_{t,\tau}, \nu_{t,\tau})(dx) \end{aligned}$$

where

$$\begin{aligned} \mu_{t,\tau} &= \Phi^{-1}(A_t/\pi) \sqrt{\left(\frac{T-t}{T-\tau}\right)^\beta} \\ \nu_{t,\tau} &= \left(\frac{T-t}{T-\tau}\right)^\beta - 1. \end{aligned}$$

Price Dependence on T and Sensitivity to β

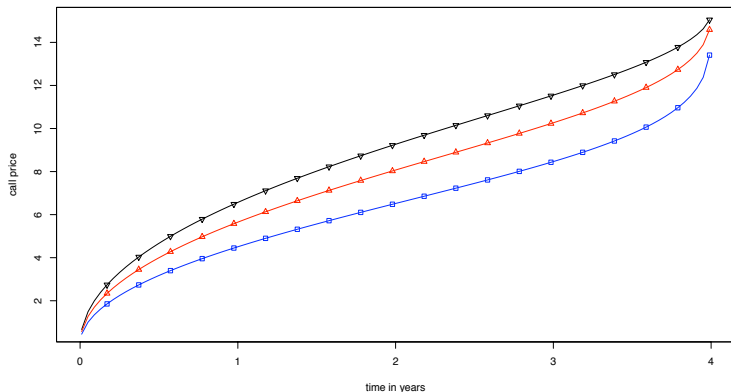
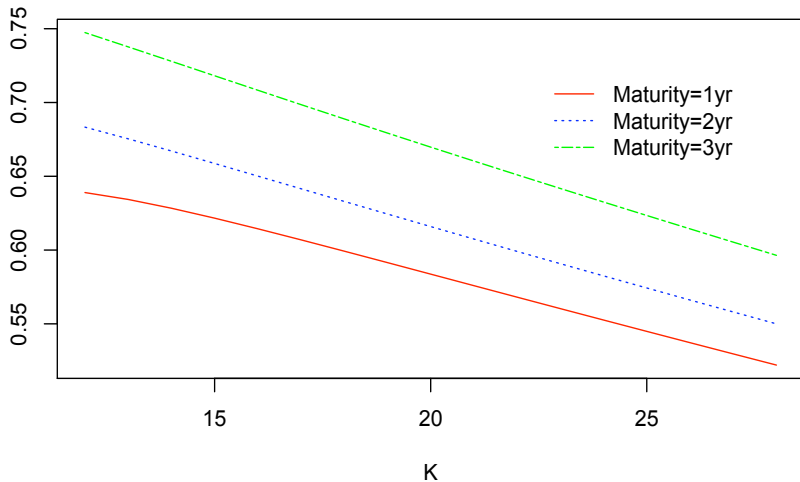
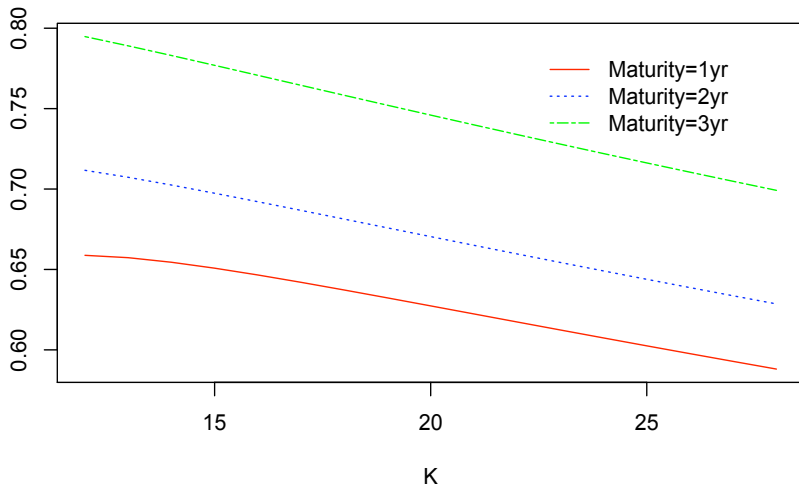


Figure: Dependence $\tau \mapsto C_0(\tau)$ of Call prices on maturity τ . Graphs $\square$, $\triangle$, and ∇ correspond to $\beta = 0.5$, $\beta = 0.8$, $\beta = 1.1$.

Implied Volatilities for Different Maturities

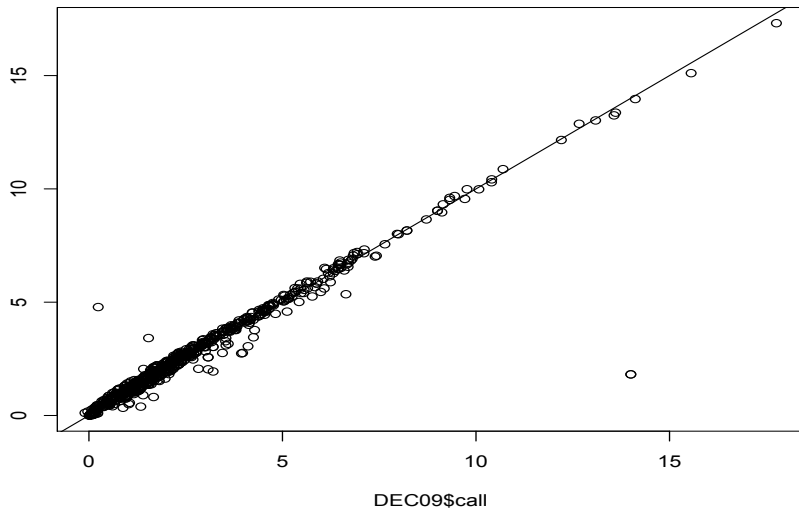


Implied Volatilities for Different Maturities

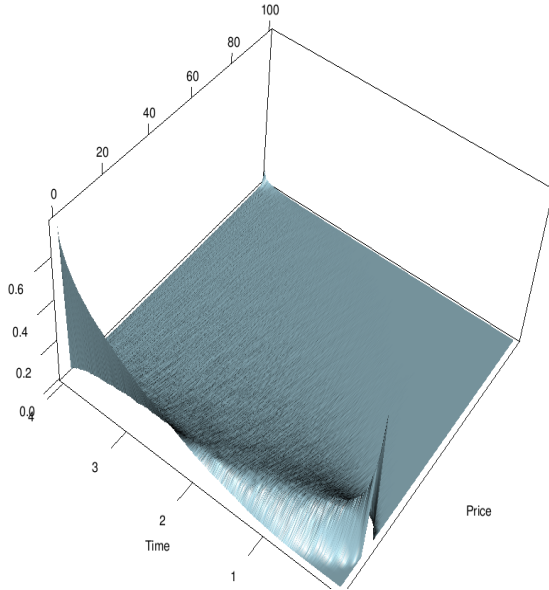


More Calibration away from End of Period

One-Period Formula Prices vs Dec09 Quotes, $\beta=0.5$



Risk Neutral Densities



Presentations based on 2) & 4)

- 1 R.C., M. Fehr and J. Hinz: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM J. Control and Optimization* (2009)
- 2 R.C., M. Fehr, J. Hinz and A. Porchet: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM Review* (2009)
- 3 R.C., M. Fehr and J. Hinz: Properly Designed Emissions Trading Schemes do Work! (working paper)
- 4 R.C., and J. Hinz: Risk-Neutral Modeling of Emission Allowance Prices and Option Valuation (working paper)
- 5 R.C. & M. Fehr: Auctions and Relative Allocation Mechanisms for Cap-and-Trade Schemes (working paper)
- 6 R.C. & M. Fehr: The Clean Development Mechanism: a Mathematical Model. (submitted *Proc. 2008 Ascona Conf.*)