Pricing Game Options with Call Protection

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In this work we deal with the issue of pricing numerically by simulation convertible bonds, or more general game options. A convertible bond can be seen as a coupon-paying and callable American option. Call times are typically subject to constraints, called call protections, preventing the issuer from calling the bond at certain sub-periods of time. Moreover the nature of the call protection may be very path-dependent, which leads, after extension of the state space to markovianize the problem, to highly-dimensional pricing problems. Deterministic pricing schemes are then ruled out by the curse of dimensionality, and simulation methods appear to be the only viable alternative.

We thus consider in this work problems corresponding to more and more complex, yet commonly encountered in practice, clauses of call protection. We propose in each case a reference, but heavy, if practical, deterministic pricing scheme, as well as a more efficient (as soon as the problem dimension exceeds a few units) and practical Monte Carlo simulation/regression pricing scheme.

In each case we derive the pricing equation; using advanced reflected BSDE discretization techniques, we establish the convergence of the Monte Carlo simulation/regression scheme, and we illustrate our results by numerical experiments. One thus gets a practical and mathematically justified approach to the problem of pricing by simulation convertible bonds with highly path-dependent call protection. More generally, this work is an illustration of the real abilities of simulation/regression numerical schemes for high to very high-dimensional pricing problems (up to a dimension of 30 in the context of the application at hand of this paper).