Pricing and hedging in carbon emissions markets

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Global warming and climate change

- Global warming and its dangerous consequences have gained increased public attention in recent years.
- Sustained economic growth since the industrial revolution has gone hand in hand with an increased burning of fossil fuels such as coal, oil and gas in a chemical process that releases carbon dioxide (CO_2), one of the most abundant greenhouse gases.
- For various policy studies on the potential impacts of climate change and options for adaptation and mitigation, see the website (`http://www.ipcc.ch/`) of *Intergovernmental Panel on Climate Change*.

Kyoto protocol

- The Kyoto protocol opened for signature at the 1997 conference in Kyoto, Japan.
- Signatory nations must reduce their emissions for carbon dioxide and five other gases in 2008-2012 by 5% with respect to 1990 levels in order to comply with the protocol.
- The European Commission (EC) launched the European Climate Change Programme (ECCP) in June 2000 with the objective to identify, develop and implement the essential elements of an EU strategy to implement the Kyoto Protocol.
- The European Union Emission Trading Scheme (EU ETS) is a significant part of the ECCP and currently constitutes the largest emissions trading scheme in the world.

EU ETS and carbon credits

- To participate in the ETS, EU member states must first submit a National Allocation Plan (NAP) for approval to the EC.
- Selected carbon intensive installations such as steel manufacturers, power stations of above 20 MW capacity, cement factories, etc. receive free emission credits under the terms of this NAP, enabling them to emit greenhouse gases up to the assigned tonnage.
- Installations can bilaterally trade emission certificates under the EU ETS, in order to offset any excess or shortage of carbon emission credits above NAP limits.
- About 12.000 installations within the Union are covered by the EU ETS in a first phase (2005-2007), representing almost 50% of total carbon emissions.

Trading within ETS

- Actual trading with EU ETS emission allowances began January 1st, 2005.
- By the end of the same year, almost 400 million tonnes of carbon equivalent had been traded.
- During Phase I (2005-2008) industries included in the EU ETS could opt to carry on their short position to the next year provided a fine of €40 per tonne is paid. This penalty has increased to €100 in Phase II (2008-2012)
- Prices for Phase I contracts at the end of the first phase will thus be nonzero only if the EU zone is net short EU ETS carbon credits for this phase.
- Consequently, the impact of the release of sensitive information regarding the ETS net position in carbon emission allowances can be dramatic, as it was illustrated in April 2006.

A price history

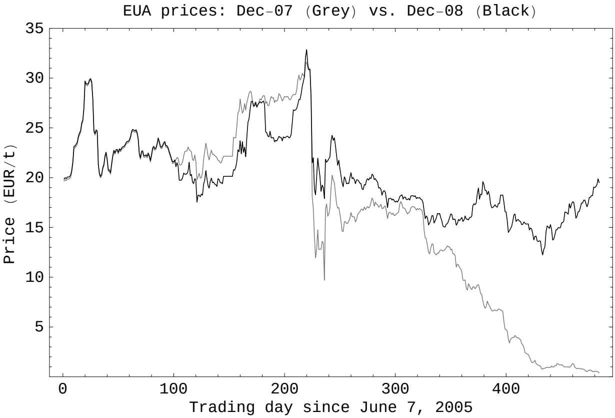

Figure: EUA price history between June 7th, 2005 and May 5th, 2007

Model

- We aim to obtain a relationship between the spot (the permit that matures at the end of the current year) and the forward (the permit that matures at the end of the next year) prices of carbon permits.
- It is apparent from the earlier figure that the EUA prices before and after the information release have different drifts.
- Moreover, starting from the last quarter of 2006 Dec-07 prices and Dec-08 prices show opposite trends.
- The reason for this behaviour is the shift in the demand for worthless spot (henceforth called EUA0) contracts to forward (henceforth called EUA1) contracts.
- In view of these observations we model the forward prices process, denoted with S, as follows:

$$dS_t = S_t(\mu + \alpha\theta_t)dt + \sigma S_t dW_t, \tag{1}$$

with $S_0 = s$.

- Here, θ is a Markov chain modeling the net position of the market, and W is a Brownian motion independent of θ.
- θ is a càdlàg Markov chain in continuous time taking values in $E := \{-1, 1\}$. $\theta_t = 1$ (resp. $\theta_t = -1$) corresponds to market being long (resp. short) at time t.
- The assumption that θ takes only two values is for simplicity and our theory can be extended to the case when E is any finite set.
- We suppose the Markov chain θ stays in state i for an exponential amount of time with parameter $\lambda(i)$.
- The initial distribution for θ is denoted with p.

Pricing with no banking of the unused permits

- The following assumption is to simplify the computations and the exposition. We stress here that our approach still works without the next assumption.

Assumption 1

$\lambda(1) = \lambda(2) = \lambda.$

- Letting P denote the price process of EUA0 contracts, we have the following relation between S and P at time T

$$P_T = \begin{cases} S_T + K, & \text{if } \theta_T \leq 0; \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

- Next we will discuss the *local risk minimisation* approach to the pricing and hedging in EU ETS market under complete and incomplete information regarding the net position of the zone.

Pricing under complete information

- For the time being let's suppose the market has full information on θ. Note that the market is still incomplete since θ is not tradable.
- Following Föllmer and Schweizer (1991) we *define* the *optimal* hedging strategy for a given contingent claim in an incomplete market as follows.

Definition 1

Let $C \in L^2(\Omega, \mathcal{F}_T, \mathbb{P})$ be a contingent claim. A predictable trading strategy ξ^C is said to be optimal if there exists a square integrable $\mathbb{F}$-martingale, L^C, orthogonal to W such that

$$C = c + \int_0^T \xi_t^C dS_t + L_T^C. \tag{3}$$

- Existence of (3) is intimately linked to the so-called *minimal* martingale measure.

Minimal martingale measure

Definition 2

Let X be a continuous semimartingale with the canonical decomposition $X = X_0 + M + A$ with M a martingale and A is adapted, continuous and of finite variation. A probability measure $\widehat{\mathbb{P}} \sim \mathbb{P}$ is called *minimal martingale measure* if X follows a martingale under $\widehat{\mathbb{P}}$, $\widehat{\mathbb{P}} = \mathbb{P}$ on $\mathcal{F}_0$ and any square integrable martingale orthogonal to M remains a martingale under $\widehat{\mathbb{P}}$.

The minimal martingale measure is uniquely determined and in our case is defined by

$$\frac{d\widehat{\mathbb{P}}}{d\mathbb{P}} = \exp\left(-\int_0^T \frac{\mu + \alpha\theta_s}{\sigma} dW_s - \frac{1}{2}\int_0^T \left(\frac{\mu + \alpha\theta_s}{\sigma}\right)^2 ds\right). \tag{4}$$

Pricing under the minimal martingale measure

Definition 3

Let $C \in L^2(\Omega, \mathcal{F}_T, \mathbb{P})$ be a contingent claim and let $\widehat{\mathbb{P}}$ be the unique minimal martingale measure for S given by (4). The fair price P_t^C at time t for C is given by

$$P_t^C := \widehat{\mathbb{E}}[C|\mathcal{F}_t].$$

- For the problem under consideration $C = (S_T + K)\mathbf{1}_{\{\theta_T = -1\}} = \frac{S_T + K}{2}(1 - \theta_T)$.
- Recall that $\theta = \theta_0 + N + A$ where N is a martingale and A is predictable process.
- Since θ and W are independent, it follows that N and W are orthogonal martingales under $\mathbb{P}$; thus, N remains a martingale under $\widehat{\mathbb{P}}$ and is orthogonal to S.

Pricing carbon permits under full information

Proposition 1

Let

$$M_t := \theta_t \exp(-2\lambda(T-t)), \qquad t \in [0,T].$$

Then M is a $\widehat{\mathbb{P}}$-martingale and

$$\widehat{\mathbb{E}}_t\left[\frac{S_T+K}{2}A_T\right] = \theta_t \frac{S_t+K}{2}\exp(-2\lambda(T-t)) - (\theta_0+N_t)\frac{S_t+K}{2}.$$

Theorem 4

The fair price for EUA0 contracts is given by

$$P_t = (S_t + K)\frac{1 - \theta_t \exp(-2\lambda(T - t))}{2}.$$

The optimal hedging strategy, ξ^0 associated with EUA0 contracts is given by

$$\xi_t^0 := \frac{1 - M_t}{2},$$

for each $t \in [0, T]$.

In other words, part of the risk at time t, corresponding to the term $\int_0^t \frac{1-M_s}{2} dS_s$, can be hedged if one follows the *locally-risk minimising strategy* which consist of holding $(1 - M)/2$ shares of the traded underlying, whose price process is given by S.

Pricing under incomplete information

Now we study the pricing of EUA0 contracts under incomplete information. We suppose the only information available to the market is the usual right-continous and complete augmentation of S, denoted with $\mathcal{F}^S$ and the one-time announcement of the true value of θ at time T. If $\mathcal{G}$ denotes the filtration modelling the information structure of the market, then

$$\mathcal{G}_t = \begin{cases} \mathcal{F}_t^S, & \text{for } t < T; \\ \mathcal{F}_T^S \vee \sigma(\theta_T), & \text{for } t = T. \end{cases}$$

Let $\overline{\theta}$ denote the optional projection of θ to $\mathcal{F}^S$ which gives $\overline{\theta}_t = \mathbb{E}[\theta_t | \mathcal{F}_t^S]$, for each $t \geq 0$. We now apply the aforementioned local-risk minimisation approach to the pricing and hedging of EUA0 under incomplete information, i.e. when the available information is modelled by $\mathcal{G}$.

Theorem 5

Define $\overline{W}$ by

$$\overline{W}_t = \int_0^t \frac{dS_s - (\mu + \alpha\overline{\theta}_s)S_s ds}{\sigma S_s},$$

and Z by $Z_t = \mathbf{1}_{[t=T]}(\theta_T - \overline{\theta}_t)$ for each $t \geq 0$. Then, $\overline{W}$ and Z are orthogonal $\mathcal{G}$-martingales. Moreover, $\overline{W}$ is a $\mathcal{G}$-Brownian motion.

Definition 6

The fair price of the EUA0 contracts at time t under incomplete information is defined to be

$$\overline{P}_t := \mathbb{E}^*[(S_T + K)(1 - \theta_T)/2|\mathcal{G}_t],$$

where $\mathbb{E}^*$ is the expectation operator under $\mathbb{P}^*$, the unique minimal martingale measure associated to S with respect to $\mathcal{G}$.

- No explicit formulae can be found for pricing or hedging.
- One can do Monte-Carlo simulation using the conditional probabilities obtained by filtering methods, or obtain a PDE for the pricing function.

Calculation of short-probabilities

It is possible to calculate the probabilities related to the zone's net position. Let $\pi_i(t) := \mathbb{P}[\theta_t = i|\mathcal{G}_t]$ for each $i \in \{-1, 1\}$. Clearly, $\overline{\theta}_t = \pi_1(t) - \pi_{-1}(t) = 2\pi_1(t) - 1$. Therefore,

$$\begin{aligned}\pi_i(t) &= \mathbb{P}[\theta_0 = i|\mathcal{G}_0] + \int_0^t (\lambda\pi_j(s) - \lambda\pi_i(s))\, ds \\ &+ \int_0^t \pi_i(s)\frac{(\mu + \alpha i)S_s - S_s(\{\mu + \alpha i\}\pi_i(s) + \{\mu + \alpha j\}\pi_j(s))}{\sigma S_s} d\overline{W}_s,\end{aligned}$$

where $\overline{W}$ is the $\mathcal{G}$-Brownian motion defined in Theorem 5, and $\{j\} = E\backslash\{i\}$.

Since $\pi_i(s) + \pi_j(s) = 1$ for all s, the above reduces to

$$\begin{aligned}\pi_i(t) &= \mathbb{P}[\theta_0 = i|\mathcal{G}_0] + \int_0^t \lambda\,(1 - 2\pi_i(s))\,ds \\ &\quad + \int_0^t \pi_i(s)\frac{\alpha(i-j)(1-\pi_i(s))}{\sigma}d\overline{W}_s.\end{aligned}$$

In particular, for $i = 1$,

$$\pi_1(t) = p + \int_0^t \lambda\,(1 - 2\pi_1(s))\,ds + \int_0^t \pi_1(s)\frac{2\alpha(1-\pi_1(s))}{\sigma}d\overline{W}_s. \quad (5)$$

Effect of intermediate announcements

- Every year, the European union aggregates submitted emission data and compares this to the quantity of allowances surrendered.
- The processing of emissions data for the entire zone usually takes a couple of months time, and announcements on the zone's net position are not released until mid April every year. Until that time, spot trading in the contract for delivery in the past year still takes place.
- As the EU so announces net results every year, we must look a bit deeper into the effects of intermediate announcements on net positions.

- To be more concrete, suppose at some $t_0 < T$ the true position of the zone is revealed to the market. To ease the calculations we further assume that there will be *no further announcements* before time T.
- Now, we redefine $\overline{\theta}$ so that $\overline{\theta}_t = \mathbb{E}[\theta_t | \mathcal{F}_t^S, \theta_{t_0}]$ for $t \geq t_0$. This implies that the dynamics of $\overline{\theta}$ changes to

$$\overline{\theta}_t = \theta_{t_0} - 2\int_{t_0}^{t} \lambda \overline{\theta}_s ds + \frac{\alpha}{\sigma}(1 - \overline{\theta}_s{}^2) d\overline{W}_s, \tag{6}$$

for $t \geq t_0$.
- Therefore, typically, there will be a jump in $\overline{\theta}$ at time t_0 since $\overline{\theta}_{t_0-}$ will be different than θ_{t_0} as long as $0 < \mathbb{P}(\theta_{t_0} = 1) < 1$.

Pricing under intermediate announcements

Theorem 7

Suppose that the true state of θ is revealed at time $t_0 < T$. The fair price of EUA0 contracts is given by

$$P_t = \begin{cases} h(t, S_t, \overline{\theta}_t) - Z_t \frac{S_t + K}{2}, & \text{for } t \geq t_0, \\ Z_t^h + h(t, S_t, \overline{\theta}_t), & \text{for } t < t_0, \end{cases}$$

where h is a continuous solution of a certain PDE and

$$Z_t^h = \mathbb{E}^*[h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0}, \overline{\theta}_{t_0-}) | \mathcal{F}_t^S],$$

for $t \leq t_0$. P has a jump at t_0 and the jump size equals

$$\Delta P_{t_0} = h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0}, \overline{\theta}_{t_0-}) - Z_{t_0}^h.$$

Introducing banking

- Since the beginning of Phase II all unused carbon permits can be banked for future use. Therefore, the spot contracts are not necessarily worthless even if the zone is net long at their maturities.
- This necessitates a modification of the spot-forward relationship that we introduced earlier.
- If one assumes risk-neutral trader the correct relationship is then

$$P_T = \begin{cases} S_T + K, & \text{if } \theta_T = -1, \\ \mathbb{E}[\mathbf{1}_{[\theta_{2T}=-1]} S_{2T} | S_T, \theta_T = 1] & \text{if } \theta_T = 1. \end{cases},$$

  where S_{2T} is time-2T price of the forward contract that matures at time 2T.
- In the next slides we present the results of various numerical studies when banking is in effect.

Numerical studies

The constants in the following plots are taken from the the table below.

Parameter	Value
α	-2
σ	0.5
λ_1	160
λ_{-1}	30
K	100
μ	0.5
S_0	30.00
θ_0	1
p	1
T	1

- We set $T = 1$ and divide this period into 430 trading dates of equal length. We had in mind the price evolution during June 2006-Dec 2007 from which we estimated the above parameters using an EM algorithm.
- We opted for a scenario where the true state of the market is revealed at $t = 0.85$.

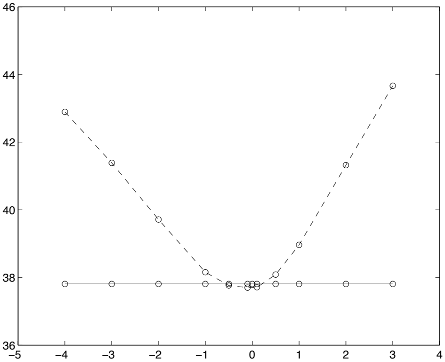

Figure: Spot prices as a function of α. Solid line represents the price under full information and dashed line under incomplete information.

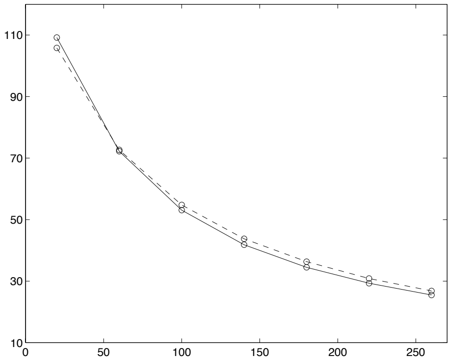

Figure: Spot prices as a function of λ_1. Solid line represents the price under full information and dashed line under incomplete information.

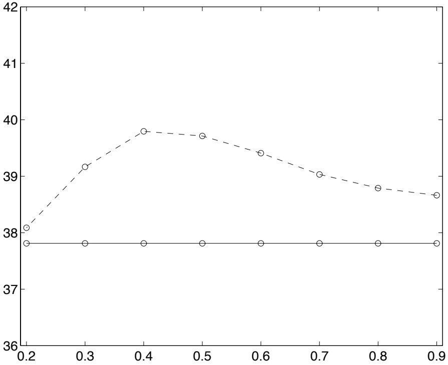

Figure: Spot prices as a function of σ. Solid line represents the price under full information and dashed line under incomplete information.

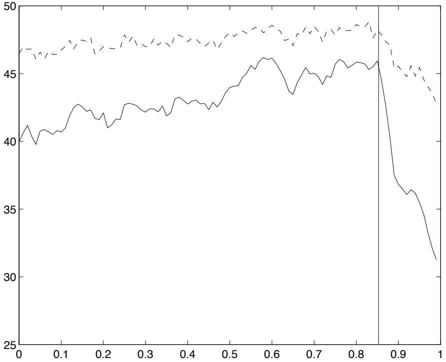

Figure: Evolution of spot and forward prices. Solid line represents the forward price and dashed line the spot price. Vertical line represents the time

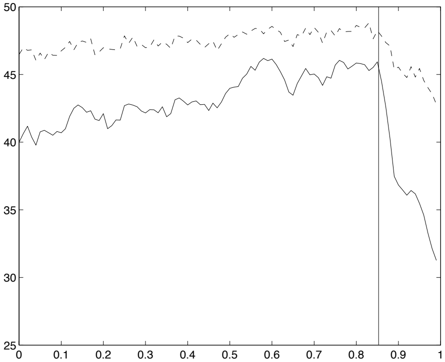

Figure: Evolution of spot and forward prices. Solid line represents the forward price and dashed line the spot price. Vertical line represents the time

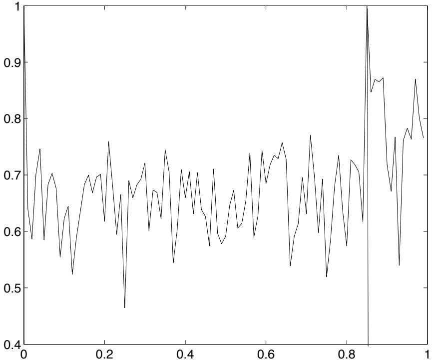

Figure: Projection of θ onto market's filtration.

Conclusions and extensions

- We discussed the pricing and hedging of EUA contracts traded within the EU ETS scheme.
- One can come up with explicit formulas for pricing and hedging under the assumption that the market's net position is common knowledge among the market participants.
- Under the more realistic setting where the market does not observe the net position directly, the price can obtained as a solution to a boundary value problem or via Monte Carlo simulation. The numerical solution for prices can be obtained very fast.
- We can also incorporate intermediate announcements on the true state of zone's net position.

- For a better calibration to market data one might consider a Markov chain with more than two states when modeling θ.
- Instead of using a Markov chain for θ, one can use a Gaussian process. Then one can use linear filtering for pricing purposes. This is done in the thesis of Takeshi Yamada.