A Coupling Interface Method for Elliptic Complex Interface Problems *

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Abstract.

We propose a coupling interface method (CIM) under Cartesian grid for solving elliptic *complex* interface problems in arbitrary dimensions, where the coefficients, the source terms and the solutions may be discontinuous or singular across the interfaces. It is a dimension-by-dimension approach. It consisits of a first-order version (CIM1) and a second-order version (CIM2).

In one dimension, the CIM1 is derived based on a linear approximation on both sides of the interface. The method is extended to high dimensions through a dimension-by-dimension approach. To connect information from each dimension, a coupled equation for the first derivatives is derived through the jump conditions in each coordinate direction. The resulting stencil uses the standard 5 grid points only in two dimensions. Similarly, the CIM2 is derived based on a quadratic approximation in each dimension. In high dimensions, a coupled equation for the principal second derivatives $u_{x_k x_k}$ is derived through the jump conditions in each coordinate direction. The cross derivatives are approximated by one-side interpolation. This approach reduces the number of grid points needed for one-side interpolation. The resulting stencil involves 8 grid points in two dimensions and 12-14 grid points in three dimensions.

The CIM1 requires that the interface intersects each grid segment (the segment connecting two adjacent grid points) at most once. This is a very mild restriction and can always be achievable by refining meshes. The CIM2 requires basically that the interface does not intersect two adjacent grid segments simutaneously. In practice, we classify the underlying Cartesian grid points into interiors, normal on-fronts and exceptionals, where a standard five-point central finite difference method, the CIM2 and the CIM1 are adopted, respectively. This method maintains second order accuracy in most applications due to the fact that usually the number of normal on-front grid points is $O(h^{1-d})$ and the number of the exceptional points is $O(1)$. Here, d is the dimension and h is the mesh size. As a result, it is robust and flexible to handle complex interface

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problems.

A numerical study for the condition number of the resulting linear system in one dimension has been performed. It is shown that the resulting linear system has the same behavior as that of the discrete Laplacian, independent of the constrast of the coefficient and discretization. Further, we also give a proof of the solvability of the coefficient for the coupling systems.

A comparison study with other interface methods is performed. Algebraic multigrid method is employed to solve the resulting linear system. Numerical tests demonstrate that CIM is very stable and second order in the maximal norm for both u and its gradients with less errors as compared with other methods. In addition, this method passes many tests of complex interface problems in two and three dimensions. Therefore, we believe that it is a competitive method.