Subcell shock capturing with high order discontinuous Galerkin method

Vinh-Tan Nguyen

Singapore-MIT Alliance

E4-04-10, 4 Engineering Drive 3, Singapore 117576

smanvt@nus.edu.sg

Abstract

A shock capturing scheme is proposed with discontinuous Galerkin methods appearing to provide high order accurate approximations on arbitrary unstructured grids. An effective strategy inspired in the classical artificial viscosity approach developed by Neumann and Richmyer is introduced. Instead of employing the inherent dissipation obtained from discontinuous Galerkin discretizations, an amount of artificial viscosity is explicitly added to the system with the purpose of eliminating high frequencies in solutions which can develop discontinuities over time. The shock is thus spread over a length scale such that it can be well approximated in the space of the approximating polynomials. The viscosity is only added to the elements in the shock region flagged by the sensor obtaining from the decomposition of the solution in the space of the Koornwinder orthogonal basis. Several models including the Laplacian and physically motivated artificial viscosity are presented and shown to be able to obtain subcell resolutions for the shocks. The method also gives an accuracy of $\mathcal{O}(h/p)$ for the solu-

tion in the vicinity of the shocks with the approximation order of p on the grid of size h. Several applications in one and two dimensions are presented to exemplify the advantages of the proposed scheme.
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