

Title: C^∞ Symmetric Tight Wavelet Frames and Nonstationary Cascade Algorithms

Abstract: Motivated by the interesting work of Cohen-Dyn on nonstationary orthonormal wavelets, in this talk, we shall discuss nonstationary tight wavelet frames and convergence of nonstationary cascade algorithms in $L_2(\mathbb{R})$. We present a general algorithm for constructing nonstationary tight wavelet frames and show that there are compactly supported C^∞ symmetric tight wavelet frames with the spectral frame approximation order. Furthermore, we present a family of symmetric compactly supported C^∞ orthonormal complex wavelets in $L_2(\mathbb{R})$. A comprehensive analysis of nonstationary tight wavelet frames and orthonormal wavelet bases in $L_2(\mathbb{R})$ is given. We show that a Sobolev space of an arbitrary fixed order of smoothness can be characterized in terms of the weighted ℓ_2 -norm of the analysis wavelet coefficient sequences using a fixed compactly supported nonstationary tight wavelet frame in $L_2(\mathbb{R})$ derived from masks of pseudo-splines. Therefore, our constructed compactly supported nonstationary tight wavelet frames of $L_2(\mathbb{R})$ can be properly normalized into a pair of dual wavelet frames in any Sobolev space. This talk is based on [B. Han and Z. Shen, Compactly supported symmetric C^∞ wavelets with spectral approximation order, *SIAM J. Math. Anal.*, to appear] and [B. Han and Z. Shen, Characterization of Sobolev spaces of arbitrary smoothness using nonstationary tight wavelet frames, *Israel J. Math.*, to appear].