Title: MRA Framelets and Their Connections to Riesz Wavelets.

Abstract: Redundant wavelet systems such as tight framelets and bi-frames have many applications in signal denoising and image processing. Recently, a general method, called the Oblique Extension Principle (OEP), has been proposed to derive tight framelets and bi-frames from refinable functions. In this talk, we shall first discuss the OEP method and the approximation properties of framelets. Then we shall discuss several applications of OEP for constructing univariate tight framelets and bi-frames. Using OEP, we show that tight framelets and bi-frames can be easily constructed from refinable functions. For example, we can show that from any *symmetric stable* refinable function with a finitely supported mask, one can always derive a *symmetric* tight framelet with three compactly supported generators and highest possible order of vanishing moments. We also show that stability of the refinable function cannot be dropped for obtaining tight framelets via OEP.

On the other hand, non-redundant wavelet systems such as Riesz wavelets are of interest in numerical algorithms. For any positive integer m, B_m denotes the B-spline of order m. Define a wavelet function ψ by $\hat{\psi}(2\xi) := e^{-i\xi}(1 - e^{i\xi})^m \widehat{B_m}(\xi)$. Then we show that $\{\psi_{j,k} := 2^{j/2}\psi(2^j \cdot -k) \ : \ j, k \in \mathbb{Z}\}$ is a Riesz basis in $L_2(\mathbb{R})$. Next, we shall explore several interesting connections between framelets/bi-frames (redundant systems) and Riesz wavelets (non-redundant systems). For example, we show that one of the generators in the B-spline tight framelets obtained in the [Daubechies-Han-Ron-Shen] paper as well as many other papers indeed generates a Riesz wavelet basis. On the other hand, by adding redundancy into a Riesz wavelet system, one can always obtain an MRA bi-frame so that a fast frame transform is possible. Some examples will be provided to illustrate the results presented in this talk.

Joint works with Ingrid Daubechies, Amos Ron, Zuowei Shen and Qun Mo. Related papers are available at `http://www.ualberta.ca/~bhan`.

1. I. Daubechies, B. Han, A. Ron, and Z. W. Shen, Framelets: MRA-based constructions of wavelet frames, *Appl. Comput. Harmon. Anal.*, **14** (2003), 1–46.

2. I. Daubechies and B. Han, Pairs of dual wavelet frames from any two refinable functions, *Constr. Approx.*, **20** (2004), 325–352.

3. B. Han and Z. W. Shen, Wavelets with short support, preprint, (2003).

4. B. Han and Q. Mo, Tight wavelet frames generated by three symmetric B-spline functions with high vanishing moments, *Proc. Amer. Math. Soc.*, **132** (2004), 77-86.

5. B. Han and Q. Mo, Splitting a matrix of Laurent polynomials with symmetry and its application to symmetric framelet filter banks. *SIAM J. Matrix Anal. Appl.*, in press.

6. B. Han and Q. Mo, Symmetric MRA tight wavelet frames with three generators and high vanishing moments, preprint, (2003).