Title: Riesz Wavelets from the Loop Scheme in Computer Graphics

Abstract: In this talk, we shall discuss Riesz wavelets with short support in both 1D and 2D. As non-redundant wavelet systems, Riesz wavelets are of interest in mesh compression in computer graphics and in numerical solutions to PDEs. The Loop scheme is well-known and widely used in CAGD and computer graphics. Recently, by a method of Riemenschneider and Shen on construction of wavelets, a wavelet system has been derived from the Loop scheme (for simplicity, we called it a Loop wavelet system) and has been employed with great success in mesh compression in the pioneering work of Khodakovsky, Schröder and Sweldens for progressive geometry compression. Despite the great success of the Loop wavelet system, until now it remains open whether such Loop wavelet system is stable or not; that is, whether it is a Riesz basis or not. In this talk, we shall confirmatively answer this question by studying Riesz wavelet bases in any dimension.

On the other hand, let ϕ be a refinable function satisfying $\hat{\phi}(2\xi) = \hat{a}(\xi)\hat{\phi}(\xi)$ with the mask $\hat{a}$ being a 2π-periodic trigonometric polynomial. If the shifts of ϕ are orthonormal (that is, $\{\phi(\cdot - k) \ : \ k \in \mathbb{Z}\}$ is an orthonormal system), by defining a wavelet function ψ by $\hat{\psi}(2\xi) := e^{i\xi}\overline{\hat{a}(\xi + \pi)}\hat{\phi}(\xi)$, then ψ generates an orthogonal wavelet basis (that is, $\{\psi_{j,k} = 2^{j/2}\psi(2^j \cdot -k) \ : \ j, k \in \mathbb{Z}\}$). But many refinable functions such as B-spline functions do not have orthonormal shifts. It is of interest to ask the following question: whether ψ, as defined above, generates a Riesz wavelet basis in $L_2(\mathbb{R})$ or not? In this talk, we shall prove that for all the B-spline functions and for all the refinable functions associated with the Deslauriers-Dubuc interpolatory masks, the conjecture holds. Such results on 1D Riesz wavelets are of interest in CAGD and computer graphics: the similar wavelet systems derived from the Catmull-Clark scheme and the Kobbelt scheme are also 2D Riesz wavelet bases (Note that for the regular mesh, the Catmull-Clark scheme is the tensor-product of the B-spline of order 4 and the Kobbelt scheme is the tensor-product of the 4-point Deslauriers-Dubc scheme).

Finally, we shall shortly discuss some interesting connections between Riesz wavelets (non-redundant system) and the framelets/bi-frames (redundant systems).

The results in this talk are based on two joint papers with Zuowei Shen. Some related papers are available at `http://www.ualberta.ca/~bhan`.

1. B. Han and Z. W. Shen, Wavelets with short support, preprint, (2003).

2. B. Han and Z. W. Shen, Wavelets from the Loop scheme with applications in computer graphics, preprint, (2004).

3. I. Daubechies, B. Han, A. Ron, and Z. W. Shen, Framelets: MRA-based constructions of wavelet frames, *Appl. Comput. Harmon. Anal.*, **14** (2003), 1–46.