Adaptive Scattered Data Fitting by Spline–Wavelets: Regularization and Robust Fitting

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In [CK1], an adaptive method to approximate unorganized clouds of points by smooth surfaces based on (tensor products of boundary adapted spline–)wavelets has been described. The general fitting algorithm operates on a coarse–to–fine basis. It selects on each refinement level in a first step a reduced number of wavelets which are appropriate to represent the features of the data set. In a second step, the fitting surface is constructed as the linear combination of the wavelets which minimizes the distance to the data in a least squares sense. This is followed by a thresholding procedure on the wavelet coefficients to discard those which are too small to contribute much to the surface representation.

This strategy is then generalized to a classically regularized least square functional by adding a Sobolev norm, taking advantage of the capability of wavelets to characterize Sobolev spaces of even fractional order. After recalling the usual cross validation technique to determine the involved smoothing parameters, some examples of fitting severely irregularly distributed data, synthetically produced and of geophysical origin, are presented. In order to reduce computational costs, we then introduce a multilevel generalized cross validation technique which goes beyond the Sobolev formulation and exploits the hierarchical setting based on wavelets [CK2]. We illustrate the performance of the new strategy on some geophysical data.

Moreover, within this regularized least–squares approach, we investigate some statistical robust estimators to handle outliers in the data [CK3]. In particular, our wavelet scheme yields a numerically fast and reliable way to detect outliers which can amount to up to 15% of the total amount of data.

References

[CK1] D. Castaño and A. Kunoth. Adaptive fitting of scattered data by spline-wavelets. In: Curves and Surfaces, L.L. Schumaker et.al. (eds.), Vanderbilt University Press, 2003, pp. 65–78.

[CK2] D. Castaño and A. Kunoth. Regularization of wavelet based fitting of scattered data — Some experiments. May 2004, 14 p., to appear in Numer. Alg.

[CK3] D. Castaño and A. Kunoth. Experiments on robust regression with adaptive spline–wavelets. Maunscript, July 1004, 24 p., submitted for publication.