Biorthogonal Eigensystems of Scaling Operators

Gao Xiaojie and S. L. Lee

Department of Mathematics
National University of Singapore
2 Science Drive 2, Singapore 117543

Abstract

A scaling function ϕ generates an Appell sequence of polynomials by the generating function $e^{xz}/\widehat{\phi}(iz)$. The Appell polynomials are eigenfunctions of a linear operator, and the distributional derivatives of ϕ are the eigenfunctions of its adjoint corresponding to the same eigenvalues. They form a biorthogonal system of eigenfunctions. In particular, the Appell polynomials generated by the uniform B-spline of order N are the classical Bernoulli polynomials of order N, and when suitably normalized they converge to the Hermite polynomials as $N \to \infty$. This provides a link between scaling functions and the corresponding biorthogonal systems of eigenfunctions and the classical Gaussian function and the Hermite polynomials. We also formulate and unify both the operator and its adjoint as a conditional expectation of functions of two independent random variables and study its action on the related Appell polynomials.