

Gabor analysis, Rieffel induction and Feichtinger's algebra as a link

Abstract

We will show that Feichtinger's algebra $S_0(\mathbb{R}^d)$ (also known as modulation space $M_0^{1,1}(\mathbb{R}^d)$ and well defined for general locally compact Abelian groups) is a natural Hilbert module for quantum tori and that a result by V. Losert can be interpreted as implying that the construction of such Hilbert modules gives Feichtinger's algebra, if one considers the category of time-frequency invariant homogenous Banach spaces.

Furthermore we point out that Rieffel's work on projective modules for quantum tori can be seen as an alternative approach to Gabor analysis, emphasizing the context of C^* -algebras and Hilbert modules. As a byproduct we use the symplectic Fourier transform to do Gabor analysis over locally compact Abelian groups and consider Gabor expansions over closed (non-discrete) subgroups. At the end we state the connection between Gabor analysis and the work of A. Schwarz and Yu. Manin on quantum theta functions .