

# A recursive algorithm for nonlinear wavelet thresholding : Applications to signal and image processing

Marie Farge, LMD-CNRS, ENS Paris

Joint work with Alexandre Azzalini and Kai Schneider.

Nonlinear thresholding of wavelet coefficients has been shown to be an efficient method for denoising signals with isolated singularities corrupted with Gaussian white noise. A quasi optimal value for the threshold can be computed from the noise level using the formula  $T_D = \sigma_W \sqrt{2 \ln N}$ , where  $N$  is the number of available samples of the signal and  $\sigma_W$  is the standard deviation of the noise. However, in most situations the noise level is unknown and has to be estimated. We present an algorithm proposed in (Physics of Fluids 11(8), 1999) which evaluates the value for the threshold. It recursively approximates the standard deviation of the noise with the standard deviation of the noisy signal, computes a threshold value and performs a first split from which it extracts a better estimate of the noise. Then, it iterates this procedure using the new estimate of the noise to compute the new threshold. The iteration stops when the threshold remains unchanged from its previous value. We show that the convergence of the sequence of estimated thresholds depends on a functional of the probability density function (PDF) of the noisy signal. We also find that the sequence converges towards the theoretical value  $T_D$  provided that the wavelet representation of the signal is sufficiently sparse. We compare the results obtained for examples in 1D and 2D with the results of a standard method based on the median of the wavelet coefficients of the noisy signal at small scale. Finally, we show that the recursive algorithm gives better results than this method when applied to an experimental signal measuring the atomic density of a Bose-Einstein condensate.