FAST SUMMATION AT NONEQUISPACED KNOTS BY NFFTS

Daniel Potts
University of Lübeck, Institute of Mathematics, Wallstr. 40, D–23560 Lübeck, Germany
e-mail:potts@math.uni-luebeck.de

Gabriele Steidl
University of Mannheim, Institute of Mathematics, D–68131 Mannheim, Germany
e-mail:steidl@math.uni-mannheim.de

Abstract

The fast computation of special structured discrete sums

$$f(y_j) := \sum_{k=1}^{N} \alpha_k K(\|y_j - x_k\|_2) \quad (j = 1, \ldots, M)$$

or from the linear algebra point of view of products of vectors with special structured dense matrices is a frequently appearing task in the study of particle models, in the numerical solution of integral equations (or of partial differential equations by recasting them as integral equations) and in the approximation of functions by radial basis functions. Various algorithms, e.g., the fast multipole method, the panel clustering method, H–matrix and mosaic matrix methods, were designed to speed up the summation process.

We develop a new algorithm for the fast computation of discrete sums based on the recently developed fast Fourier transform at nonequispaced knots (NFFT). Our algorithm, in particular our regularization procedure, is simply structured and can easily be adapted to different kernels K, e.g.

$$\frac{1}{x^2},\ \frac{1}{x},\ \log x,\ x^2 \log x,\ \mathrm{e}^{-\sigma x^2},\ (x^2 + c^2)^{\pm 1/2}.$$

Our method utilizes the widely known FFT and can consequently incorporate advanced FFT implementations. We prove error estimates to obtain clues about the choice of the involved parameters and present numerical examples in one and two dimensions.