Reconstructing Multivariate Functions from Scattered Data Efficiently

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In practice, one often faces the problem of reconstructing a multivariate function from a given, finite data set. In the simplest case, such a data set consists of *data values* $f_j = f(x_j) \in \mathbb{R}$, $1 \le j \le N$, coming from an unknown function f at certain *data sites* $X = \{x_1, \dots, x_N\} \subseteq \Omega \subseteq \mathbb{R}^d$, and interpolation is the most obvious reconstruction method. In general, the data sites are *scattered* over the region Ω, having no structure at all.

Approximation by positive definite kernels tries to solve this reconstruction problem by fixing a symmetric kernel $\Phi : \Omega \times \Omega \to \mathbb{R}$. Then, the approximant is chosen from the finite space $\{\Phi(\cdot, x_j) : x_j \in X\}$. The assumption on Φ being positive definite leads to a well-posed problem.

In this talk I will focus on the following topics:

- error estimates in Sobolev spaces for reconstruction processes from scattered data,
- the construction of a fast reconstruction and evaluation algorithm,
- examples from surface reconstruction in computer-aided design and from fluid-structure-interaction in aeroelasticity.