Super-Resolution Image Restoration from Blurred Low-Resolution Images*

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ABSTRACT

In this paper, we present a technique for reconstructing a high-resolution image from a blurred image sequence. The image sequence consists of decimated, blurred and noisy versions of the high-resolution image. The high-resolution image is modeled as a Markov random field, and a maximum a posteriori estimation technique is used for image reconstruction.

We consider a block-Toeplitz-Toeplitz-block (BTTB) matrix H_i as a blurring matrix. An image sequence $\{y_i\}$ is formed by

$$y_i = H_i D z + n_i \tag{1}$$

where z is a high-resolution image, D is a decimal matrix and n_i is a noise vector. This problem can be solved by the minimization and regularization technique:

$$\min_z \{\sum_{i=1}^{p} \|H_i D z - y_i\|^2 + \alpha R(z)\} \tag{2}$$

where $R(z)$ is a function measuring regularity of z and α is a regularization parameter controlling the regularity function. The corresponding system of (2) is given by

$$\left(\sum_{i=1}^{p} D^T H_i^T H_i D + \alpha L\right) z = \sum_{i=1}^{p} D^T H_i^T y_i. \tag{3}$$

where L is Laplacian matrix. In this paper, we employ the preconditioned conjugate gradient (PCG) method with the preconditioner

$$\sum_{i=1}^{p} D^T C_i^T C_i D + \alpha L_c \tag{4}$$

to solve (3) where C_i and L_c are a block-circulant-circulant-block (BCCB) matrices. The preconditioner is not a block-circulant-circulant-block matrix, but it still has a circulant structure that allows to develop a fast algorithm [1] to find its inverse very efficiently. Numerical examples are given to illustrate the effectiveness of the method. The performance of the proposed PCG method is better than that of steepest descent method given in [2].

Keywords: Circulant matrix, Toeplitz matrix, Preconditioned Conjugate Gradient method, Fourier transform, Regularization, High-Resolution, Image Reconstruction

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