Some Probability Distributions and Underlying Stochastic Mechansims in Infectious Disease Transmission Modelling

Ping Yan
Centre for Infectious Disease Prevention and Control
Public Health Agency of Canada
Ottawa, Canada

(Lecture notes for the workshop on "Mathematical Modelling of Infectious Diseases: Dynamics and Control" at Institute for Mathematical Sciences, National University of Singapore, 2005)

ABSTRACT When an infectious agent enters a susceptible population of size n, with probability π, the outbreak of a disease terminates with few cases , of which, the average number remains constant as $n \to \infty$ and the outbreak size as a proportion, f, concentrates at zero. This quantifies a "*minor outbreak*". With probability $1 - \pi$, the initial growth of infected individuals over time t may be approximated by an exponential function Ce^{rt} with rate r and the outbreak size as a number, nf, scales linearly with n, while $f > 0$ is a proportionality constant. This quantifies a "*major outbreak*". N is the random number of infections produced by an infective individual throughout its infectious period. $R_0 = E[N]$ is the basic reproductive number. In many classical infectious disease modelling literature, under the assumptions that the contacts are homogeneous(Poisson); the probability per contact is homogeneous (Bernoulli); and the infectious period is exponentially distributed or treated as fixed constant, R_0 has shown to be the single most important parameter that addresses many important public health questions. In general, the exact distribution $\Pr\{N = n\}$ needs to be specified to address some public health questions such as the risk of major outbreaks $1 - \pi$ and the distributions of size and duration of minor outbreaks. However, the same distribution $\Pr\{N = n\}$ may arise from many different combinations of stochastic mechanisms, depending on the point process for contacts, the transmission probability and the infectious period distribution. Some important quantities, such as the intrinsic growth rate r during the early phase of an outbreak, depend on these underlying stochastic mechanisms. This lecture note attempts to catalogue types of public questions that different types questions require different levels of in-depth of knowledge into the stochastic mechanism.