Decidability and Definability in the Σ_2^0 -Enumeration Degrees My First Beamer Talk

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Introduction Justification Basic Definitions and Global Theorems $\Sigma_2^0\text{-}\text{enumeration}$ Degrees

Enumeration Degrees

- Introduced by Friedberg and Rogers in 1959.
- A is enumeration reducible to B (A ≤_e B) if we can enumerate A given any enumeration of B.

Definition

 $A \leq_e B$ iff there is c.e. set Φ such that $A = \Phi^B = \{x : \exists \langle x, P \rangle \in \Phi \ (P \text{ finite and } P \subseteq B)\}$

• Similar to Turing reducibility: $A \leq_T B$ iff for $\forall x \ A(x) = \Phi^B(x) = y$ And $\Phi^B(x) = y$ iff $\exists \langle x, y, P, N \rangle \in \Phi$ ($P \subseteq B$ and $N \subseteq \overline{B}$).

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Applications

- Analysis of Types in Effective Model Theory.
- Existentially Closed Groups in Computable Group Theory.
- Computable Analysis.
- And much, much more!!!

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Introduction Justification Basic Definitions and Global Theorems $\Sigma_2^0\text{-}enumeration Degrees}$

Enumeration Degrees: Global Structure

Notation

 \mathfrak{D}_e is the set of all e-degrees. \mathfrak{D}_T is the set of all Turing-degrees.

Fact

Minimal element is $\mathbf{0}_e$ = the set of all c.e. sets.

Definition

```
\deg_e(A) \lor \deg_e(B) =_{\mathsf{def}} \deg_e(A \oplus B)
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Theorem (Case, 1971)

Every countable ideal in \mathfrak{D}_e has an exact pair.

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\mathfrak{D}_T as a Substructure of \mathfrak{D}_e .

Definition

Define the embedding $\iota : \mathfrak{D}_T \hookrightarrow \mathfrak{D}_e$ by $\iota : \deg_T(A) \mapsto \deg_e(A \oplus \overline{A}).$

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 ι is an order preserving map: $A \leq_T B \Leftrightarrow A \oplus \overline{A} \leq_e B \oplus \overline{B}$.

Definition

range(ι) = TOT = the set of total enumeration degrees.

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Facts about \mathfrak{D}_e

Theorem (Gutteridge, 1971)

 \mathfrak{D}_e has no minimal elements.

Definition

A degree **a** is quasi-minimal if $\mathbf{a} > \mathbf{0}_e$ and for all $\mathbf{b} \in \text{TOT}$, $\mathbf{b} \le \mathbf{a}$ implies $\mathbf{b} = \mathbf{0}_e$.

Theorem (Cooper, 1989)

 \mathfrak{D}_e is not dense.

Theorem (Calhoun, Slaman, 1996)

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Σ_2^0 -Enumeration Degrees

Definition

We define the enumeration jump as follows:

•
$$K_A =_{\mathsf{def}} \{ x : x \in \Phi_x^A \}$$

•
$$J(A) = K_A \oplus \overline{K_A}$$

•
$$\mathbf{0}'_{e} = \deg_{e}(J(\emptyset)) = \deg_{e}(\overline{K})$$

Theorem (Cooper, 1984)

A is Σ_2^0 iff $A \leq_e J(\emptyset)$.

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More Background Big Definitions Nies Transfer Lemma

Undecidability of Complete Theories

Given a theory, we can ask the following questions:

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Is the theory undecidable?

Question

What is the *n*, if any, s.t. the Π_n -theory is decidable and the Π_{n+1} -theory is undecidable?

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Example

Theorem

The Π_1 -theory of Σ^0_2 -enumeration degrees decidable (in the language of $\{\leq\}).$

Proof.

- Σ_1 sentence ψ is of the form $\exists \overline{x} \varphi(\overline{x})$.
- $\varphi(\overline{x})$ describes an ordering on \overline{x} .
- Can embed any finite p.o. in the Σ_2^0 -enumeration degrees.
- If $\varphi(\overline{x})$ describes a p.o., ψ is true, otherwise $\neg \psi$ is true.

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Hereditarily Undecidable

Definition

Sets *A* and *B* are *computably inseparable* if there is no computable set *C* with $A \subseteq C$ and $B \cap C = \emptyset$.

Definition

A set *S* of first order sentences is *Hereditarily Undecidable* if \overline{S} and $S \cap V$ are computably inseparable (*V* all valid sentences in language of *S*).

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Elementary Definability

Given classes of structures C, D in relational languages \mathcal{L}_{C} and \mathcal{L}_{D} , without equality, we make the following definitions:

Definition

- A Σ_k-scheme s(p̄) for L_C, in L_D, consists of Σ_k-formulas in L_D that code the universe, each relation R ∈ L_C and the negation of R.
- (a) $\alpha(\overline{p})$ is a Π_{k+1} -correctness condition for *s* if it codes #1 in \mathcal{L}_D .
- *C* is Σ_k -elementary definable w/ parameters in *D* if ∃ Σ_k -scheme *s* s.t. for all *C* ∈ *C* there is *D* ∈ *D* and $\overline{p} \in D$ s.t.

(a) $D \models \alpha(\overline{p})$ (b) $C \cong \tilde{C}$ structure in D defined by s and \overline{p} .

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- ③ C is Σ_k-elementary definable w/ parameters in D if ∃ Σ_k-scheme s s.t. for all C ∈ C there is D ∈ D and p̄ ∈ D s.t.

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- A Σ_k-scheme s(p̄) for L_C, in L_D, consists of Σ_k-formulas in L_D that code the universe, each relation R ∈ L_C and the negation of R.
- 2 $\alpha(\overline{p})$ is a Π_{k+1} -correctness condition for *s* if it codes #1 in \mathcal{L}_D .
- 3 *C* is Σ_k-elementary definable w/ parameters in *D* if ∃ Σ_k-scheme *s* s.t. for all *C* ∈ *C* there is *D* ∈ *D* and $\overline{p} \in D$ s.t.

(a) $D \vDash \alpha(\overline{p})$ (b) $C \cong \tilde{C}$ structure in D defined by s and \overline{p} .

More Background Big Definitions Nies Transfer Lemma

Elementary Definability

Given classes of structures C, D in relational languages \mathcal{L}_C and \mathcal{L}_D , without equality, we make the following definitions:

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More Background Big Definitions Nies Transfer Lemma

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More Background Big Definitions Nies Transfer Lemma

How to Show Theory Fragment is Undecidable

Lemma (Nies Transfer Lemma, 1996)

(For $k \ge 1, r \ge 2$.)

If C is Σ_k -elementary definable with parameters in D and the Π_{r+1} -theory of C hereditarily undecidable then the Π_{r+k} -theory of D hereditarily undecidable.

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More Background Big Definitions Nies Transfer Lemma

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More Background Big Definitions Nies Transfer Lemma

Theory of the Σ_2^0 -enumeration degrees is Undecidable

Theorem (Slaman, Woodin, 1997)

The First order theory of the Σ_2^0 -enumeration degrees in language $\{\leq\}$ is undecidable.

Proof.

- Finite Graphs are Σ_2 -elementary definable in Σ_2^0 -enumeration degrees with parameters.
- The Π₄-theory of Finite Graphs is hereditarily undecidable.
- Nies Transfer Lemma tells us that $\Pi_{3+2} = \Pi_5$ -theory is hereditarily undecidable.

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- Finite Graphs are Σ₂-elementary definable in Σ₂⁰-enumeration degrees with parameters.
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Main Theorem Requirements Grand Finale Future Research

 Π_3 -Theory of Σ_2^0 -enumeration degrees is Undecidable

To show this, need theory that is:

- Π₃-hereditarily undecidable and
- Σ_1 -elementary definable in Σ_2^0 -enumeration degrees

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Theorem (Nies, 1996)

The Σ_{2^-} (and hence Π_{3^-}) theory of the finite bipartite graphs with nonempty left and right domains in the language of one binary relation, but without equality, is hereditarily undecidable.

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Main Theorem Requirements Grand Finale Future Research

Coding the Universe I

Theorem (Ahmad, Lachlan, 1998)

(Ahmad Pair) There exists Σ_2^0 -enumeration degrees **a**, **b** such that $\mathbf{a} \not\leq \mathbf{b}$ but $\forall \mathbf{x} < \mathbf{a}, \mathbf{x} \leq \mathbf{b}$.

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Main Theorem Requirements Grand Finale Future Research

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Main Theorem Requirements Grand Finale Future Research

Coding the Universe II

Fix Domains $\mathcal{L} = \{0, \dots, n\}$ and $\mathcal{R} = \{\tilde{0}, \dots, \tilde{m}\}$. Build **a**, **b**, **a**₀, ..., **a**_n such that:

1 a
$$\leq$$
 b and a_i \leq b

② $\forall \mathbf{x} < \mathbf{a}, \, \mathbf{x} \leq \mathbf{b} \text{ or } \mathbf{x} \geq \mathbf{a}_i \text{ for some } i \in \mathcal{L}$

 $\mathbf{a}_0, \ldots, \mathbf{a}_n$ specified by \mathbf{a} and \mathbf{b} (Π_1 -formula):

 $\varphi(x) = x < a \land x \leq b \land \forall y \leq x (y \leq b \rightarrow y = x)$

• Not good enough, we need a Σ_1 -formula.

Main Theorem Requirements Grand Finale Future Research

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Fix Domains $\mathcal{L} = \{0, \dots, n\}$ and $\mathcal{R} = \{\tilde{0}, \dots, \tilde{m}\}$.

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Main Theorem Requirements Grand Finale Future Research

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Main Theorem Requirements Grand Finale Future Research

Coding the Universe III

Build degree **c** such that:

c ≤ a_i

 $e c \le \mathbf{a}_i \lor \mathbf{a}_j, i \neq j.$

Represent $i \in \mathcal{L}$ by $[\mathbf{a}_i, \mathbf{a}) - [\mathbf{c}, \mathbf{0}'_e]$ (Δ_0 -formula):

$$\varphi(x) = x < a \land x \not\leq b \land x \not\geq c$$

Remark

Each element of each domain is now represented by an equivalence class of degrees.

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Main Theorem Requirements Grand Finale Future Research

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Main Theorem Requirements Grand Finale Future Research

Coding the Edge Relationship

Construct degrees \mathbf{e}_0 , \mathbf{e}_1 s.t for $i \in \mathcal{L}$, $\tilde{i} \in \mathcal{R}$:

- $E(i, \tilde{\imath})$ iff $\mathbf{e}_0 \leq \mathbf{a}_i \lor \tilde{\mathbf{a}}_{\tilde{\imath}}$ iff $\mathbf{e}_1 \not\leq \mathbf{a}_i \lor \tilde{\mathbf{a}}_{\tilde{\imath}}$.

Allows Σ_1 -definition of edge relationship:

1 $E(x, \tilde{x}) \Leftrightarrow$ $(\exists y \leq x) \ (\exists \tilde{y} \leq \tilde{x}) \ (\exists z) \ (z \geq y \land z \geq \tilde{y} \land z \not\geq e_1)$ **2** $\neg E(x, \tilde{x}) \Leftrightarrow$ $(\exists y \leq x) \ (\exists \tilde{y} \leq \tilde{x}) \ (\exists z) \ (z \geq y \land z \geq \tilde{y} \land z \not\geq e_0)$

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Main Theorem Requirements Grand Finale Future Research

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- **2** $\neg E(i, \tilde{\imath})$ iff $\mathbf{e}_0 \not\leq \mathbf{a}_i \lor \tilde{\mathbf{a}}_{\tilde{\imath}}$ iff $\mathbf{e}_1 \leq \mathbf{a}_i \lor \tilde{\mathbf{a}}_{\tilde{\imath}}$.

Allows Σ_1 -definition of edge relationship:

$$\begin{array}{c} \bullet \quad E(x,\tilde{x}) \Leftrightarrow \\ (\exists y \leq x) \ (\exists \tilde{y} \leq \tilde{x}) \ (\exists z) \ (z \geq y \ \land \ z \geq \tilde{y} \ \land \ z \not\geq e_1) \\ \bullet \quad (\exists y \leq x) \ (\exists \tilde{y} \leq \tilde{x}) \ (\exists z) \ (z \geq y \ \land \ z \geq \tilde{y} \ \land \ z \not\geq e_0) \end{array}$$

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Main Theorem Requirements Grand Finale Future Research

Coding the Edge Relationship

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Main Theorem Requirements Grand Finale Future Research

Conclusion:

Theorem

Finite Bipartite Graphs are Σ_1 -elementary definable in Σ_2^0 -enumeration degrees with parameters (**a**, **b**, **c**, $\tilde{\mathbf{a}}, \tilde{\mathbf{b}}, \tilde{\mathbf{c}}, \mathbf{e}_0, \mathbf{e}_1$).

Corollary

The Π_3 -theory of Σ_2^0 -enumeration degrees is undecidable.

Question

Can we do better?

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Main Theorem Requirements Grand Finale Future Research

A new requirement

Lemma (Cooper, McEvoy 1985)

A set A is low if and only if $B \leq_e A$ implies that B is Δ_2^0 .

Modify the construction so that

(1) $\mathbf{a} \lor \tilde{\mathbf{a}}$ is low.

We now have that

- Parameters $\mathbf{a}, \mathbf{a}_0, \ldots, \mathbf{a}_n, \mathbf{c}, \mathbf{e}_0, \mathbf{e}_1$ are now low (Δ_2^0) .
- Parameter **b** is Δ_2^0 (but cannot be low) (Ahmad, 1998).

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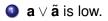
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Main Theorem Requirements Grand Finale Future Research

Corollary

 Π_3 -theory of the Δ_2^0 -enumeration degrees is undecidable.

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If \mathfrak{M} is a substructure of the enumeration degrees which contains the Δ_2^0 -degrees, then the Π_3 -theory of the \mathfrak{M} is undecidable.

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Main Theorem Requirements Grand Finale Future Research

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Main Theorem Requirements Grand Finale Future Research

Open Questions

- Is the Π₂-theory of the Σ_2^0 -enumeration degrees decidable?
- Is the Π_2 -theory of the Δ_2^0 -enumeration degrees decidable?
- Is the theory of Σ⁰₂-enumeration degrees as complex as the theory of (ω, +, ×)?
- What about decidability in other languages?

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Main Theorem Requirements Grand Finale Future Research

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The Π₂-theory Continuity of Cupping Non-splitting Degrees

The Π_2 -theory of the Σ_2^0 -enumeration degrees

We can translate a Π_2 -sentence to the question:

Given partial orders P, Q₀, ..., Q_n, with P ⊆ Q_i, does every embedding of P into the degree structure extend to an embedding of one of the Q_i?

Theorem (Lempp, Slaman, Sorbi, to appear)

Extension of Embeddings subproblem (n = 0) in Σ_2^0 -enumeration degrees is decidable.

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The Π₂-theory Continuity of Cupping Non-splitting Degrees

Continuity of Cupping

 Ahmad pointed out that if we can decide if there is an Ahmad pair whose join is complete then can decide all sentences of the form

 $\forall x \,\forall y \,\exists z \,\exists w \,\varphi(x, y, z, w).$

Theorem (Ambos-Spies, Lachlan, Soare, 1993)

If **a** and **b** are c.e. Turing degrees such that $\mathbf{a} \lor \mathbf{b} = \mathbf{0}'$, then there exists a c.e. Turing degree $\mathbf{c} < \mathbf{a}$ such that $\mathbf{c} \lor \mathbf{b} = \mathbf{0}'$.

 If this is true in the Σ⁰₂-enumeration degrees then no Ahmad pair joins to 0'_e.

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The Π₂-theory Continuity of Cupping Non-splitting Degrees

(Dis)Continuity of Cupping

Theorem

There exist Σ_2^0 -enumeration degrees **a** and **b** such that $\mathbf{a} \lor \mathbf{b} = \mathbf{0}'_e$ and for all $\mathbf{c} < \mathbf{a}, \mathbf{c} \lor \mathbf{b} < \mathbf{0}'_e$. (We call **a** a minimal cupping companion for **b**.)

Remark

The set of minimal cupping companions for **b** is an antichain. Very Useful.

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The Π₂-theory Continuity of Cupping Non-splitting Degrees

Non-splitting Degrees I

Definition

a is non-splitting if for all **b**, $\mathbf{c} < \mathbf{a}$, $\mathbf{b} \lor \mathbf{c} < \mathbf{a}$.

 Used in Ahmad pairs. We need better understanding of Ahmad pairs if we wish to settle the decidability of the Π₂-theory.

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The Π₂-theory Continuity of Cupping Non-splitting Degrees

Non-splitting Degrees II

Theorem (Kent, Sorbi)

There exists a properly Σ_2^0 non-splitting degree.

Theorem (Kent, Sorbi)

Every Δ_2^0 -degree bounds a non-splitting degree.

Question

Does every Σ_2^0 -degree bound a non-splitting degree?

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