# Derived models associated to mice

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**Abstract.**
Core model inductions reaching infinitely many Woodins, or further, rely on a key step in which one passes from some kind of *hybrid mouse* to an ordinary $L[\vec{E}]$ mouse. The following conjecture is the basic test problem for our ability to translate hybrid mice into ordinary mice.

**Conjecture 0.0.1 ((Mouse Set Conjectures (MSC)))** *Assume* $\mathsf{AD}^+$*, and that there is no* $\omega_1$*-iteration strategy for a mouse with a superstrong cardinal; then*

*(I) If* $x, y$ *are countable transitive sets,* $x \subseteq y$*, and* $x$ *is ordinal definable from parameters in* $y \cup \{y\}$*, then there is an* $\omega_1$*-iterable -mouse* $\mathcal{M}$ *over* $y$ *such that* $x \in \mathcal{M}$*, and*

*(II) if* $y$ *is countable, transitive, and* $\exists A \subseteq \mathbb{R}(HC, \in, A) \models \varphi[y]$*, then there is an* $\omega_1$*-iterable mouse* $\mathcal{M}$ *over* $y$*, and a* $\lambda$ *such that*

$$\mathcal{M} \models \lambda \text{ is a limit of Woodin cardinals,}$$

*and*

$$\mathcal{M} \models \exists A \in \mathrm{Hom}_{<\lambda}((HC, \in, A) \models \varphi[y]).$$

In lectures 3 and 4, we shall prove a partial result in this direction:

**Theorem 0.1 (Neeman, Steel)** $\mathsf{MSC}$ *holds if the hypothesis is strengthened to* $\mathsf{AD}^+$ *plus there is no iteration strategy for an mouse* $\mathcal{M}$ *such that for some* $\lambda$*,*

$$\mathcal{M} \models \lambda \text{ is a limit of Woodin cardinals,}$$

*and for some* $\kappa < \lambda$*,*

$$\mathcal{M} \models \kappa \text{ is } < \lambda \text{ strong, and a limit of } < \lambda\text{-strongs.}$$

The proof leans heavily on work of Woodin.

The mouse set conjectures ask us to construct mice with given derived models. This leads naturally to the question: *what can one say about the derived model of a mouse?* This will be the focus of lectures 1 and 2. We shall show that for certain "tractable" $\mathcal{M}, \lambda$ such that $\mathcal{M}$ is a mouse and $\lambda$ is a limit of Woodins in $\mathcal{M}$, the derived model $\mathcal{D}(\mathcal{M}, \lambda)$ of $\mathcal{M}$ at $\lambda$ satisfies $\mathsf{MSC}$. We shall show that for these tractable pairs $(\mathcal{M}, \lambda)$, there is a canonical derived model of an iterate of $\mathcal{M}$ whose reals are precisely the reals in $V$. Finally, we shall investigate the *Solovay sequence* $\langle \theta_\alpha \rangle$ of various $\mathcal{D}(\mathcal{M}, \lambda)$. For example, letting $M^\sharp_{\mathrm{wdn.lim}}$ be the minimal active mouse with a Woodin limit of Woodin cardinals, we shall show

**Theorem 0.2 (Closson, Steel)** *Let $\lambda$ be the Woodin limit of Woodins in $M^\sharp_{wdn.lim}$; then*

$$\mathcal{D}(M^\sharp_{wdn.lim}, \lambda) \models \theta = \theta_\theta.$$