Degenerate Principal Series Representations of $\mathrm{U}(p,q)$

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Abstract

This is joint work with Hung-Yean Loke. Let $p > q$ and let $G = \mathrm{U}(p,q)$. Let $P = LN$ be the maximal parabolic subgroup of G with Levi subgroup $L \cong \mathrm{GL}_q(\mathbf{C}) \times \mathrm{U}(p-q)$. For $s \in \mathbf{C}$ and $\sigma \in \mathbf{Z}$, let $\chi_{s,\sigma} : \mathrm{GL}_q(\mathbf{C}) \longrightarrow \mathbf{C}^\times$ be given by

$$\chi_{s,\sigma}(a) = |\det a|^s \left(\frac{\det a}{|\det a|}\right)^\sigma .$$

Let τ_{p-q}^μ be the irreducible representation of $\mathrm{U}(p-q)$ with highest weight μ. Let $\pi_{s,\sigma,\mu}$ be the representation of P which is trivial on N and

$$\pi_{s,\sigma,\mu}|_L = \chi_{s,\sigma} \otimes \tau_{p-q}^\mu,$$

and consider the induced representation $\mathrm{Ind}_P^G \pi_{s,\sigma,\mu}$. Let $I_{p,q}(s,\sigma,\mu)$ be the Harish-Chandra module of $\mathrm{Ind}_P^G \pi_{s,\sigma,\mu}$. Except in the cases $q = p-1$ and $q = 1$, $I_{p,q}(s,\sigma,\mu)$ is not K-multiplicity free. Here $K = \mathrm{U}(p) \times \mathrm{U}(q)$ is the maximal compact subgroup of G.

In this talk, we shall describe the methods used to determine the reducibility of $I_{p,q}$ and the composition structure of $I_{p,q}$ when it is reducible.