Let U denote the enveloping algebra of a complex semisimple Lie algebra

A family of virtual (not necessarily finite dimensional) representations $\{\pi(\nu)\}_{\nu\in\Lambda}$ of U is called a *coherent family* if for every finite dimensional module F of U (in the Grothendieck group)

$$\pi(\nu) \quad F = \sum_{\mu\in\Delta(F)} m(\mu, F)\pi(\nu+\mu)$$

where the summation is over the weights $\Delta(F)$ of F and for $\mu \in \Delta(F), m(\mu, F)$ denotes the multiplicity of μ as a weight of F.

We will describe an algebraic expression (at the level of a suitable Grothendieck group) which gives the quantum analogue $\overline{\pi}$ of a U-module π for quantum groups U_λ at roots of unity, at the level of a suitable Grothendieck group. This description involves not only the original U-module but in addition the coherent family of (virtual) U-modules to which π belongs.

The family $\{\overline{\pi}(\nu)\}_{\nu\in\Lambda}$ thus constructed is itself a coherent family of virtual representations of U_λ and satisfies

$$\overline{\pi}(\ell\nu^{''}) = \widetilde{\pi}(\nu^{''}) \quad St$$

for any $\nu^{''} \in \Lambda$; here St is the Steinberg representation and $\widetilde{\pi}(*)$ denotes the pullback by the Frobenius map from U_λ to U. Under certain conditions one should expect that $\overline{\pi}(\nu)$ is represented in the Grothendieck group by a U_λ- module (and not just a virtual module) for parameters ν in a cone.

We will discuss the validity of this fact in some low rank cases.