Estimates of automorphic forms and representation theory

Joseph Bernstein

Tel Aviv University

Abstract

Let H be the upper half plane with the hyperbolic metric of constant curvature -1. We consider the natural action of the group $G = SL(2, \mathbf{R})$ on H.

Fix a lattice (discrete subgroup) $\Gamma \subset G$ and consider the Riemann surface $Y = \Gamma \setminus H$.

The Laplace-Beltrami operator Δ acts in the space of functions on Y. When Y is compact it has discrete spectrum; we denote by $\mu_1 \leq \mu_2 \leq$ ... its eigenvalues on Y and by ϕ_i the corresponding eigenfunctions. (We assume that $||\phi_i||_{L^2} = 1$.) These functions ϕ_i are usually called *automorphic functions* or *Maass forms* .

The study of automorphic functions and the corresponding eigenvalues is important in many areas of representation theory, number theory and geometry.

In my talk I will discuss several problems of estimating numbers arising from modular forms.

One of these problems is to find good estimates of the triple products of automorphic functions.

More precisely, let us fix one automorphic function, ϕ, and consider the function ϕ^2 on Y. Since ϕ^2 is not an eigenfunction, it is *not* an automorphic function.

Since $\phi^2 \in L^2(Y)$, we may consider its spectral decomposition in the basis $\{\phi_i\}$: $\phi^2 = \sum c_i \phi_i$. Here the coefficients are given by the triple product integrals: $c_i = \langle \phi^2, \phi_i \rangle = \int_X \phi \cdot \phi \cdot \overline{\phi}_i dx$.

I will explain how one can get an estimate of these triple products using representation theory of the group $SL(2, \mathbf{R})$.

I will also explain why these triple products are of interest and how they are related to the theory of L-functions.

The main idea of the method which I will try to explain is to associate to every automorphic function ϕ an automorphic representation of the group $G = SL(2, \mathbf{R})$ and study properties of this representation.