Analytic structures on representation spaces of reductive groups

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Abstract

Let G be a real reductive group and (π, G, V) be an admissible representation of G.

In the case when $G = SL(2, \mathbf{R})$ and V is a representation of (generalized) principal series we can realize V as a space of functions on a circle. This allows us to define on the space V a family of analytic structures - namely a family of Sobolev norms W_s parameterized by a real number s.

It turns out that a similar construction can be carried out for an arbitrary reductive group G, but the family of Sobolev norms which we construct naturally depends on a parameter s which lies in the dual to split Cartan algebra of G. In other words, in this case the natural family of Sobolev norms is parameterized by several real parameters instead of one.

All this has a direct analogy for p-adic reductive groups. It turns out that in this case the situation is even simpler - namely we can use Howe's method to give a direct description of the corresponding Sobolev norms W_s.