An Introduction to the Fast-MoM

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The main objective in this presentation is threefold:
(i) We demonstrate that accelerating EM computations and at the same time systematically enhancing the accuracy of the numerical results are not necessarily mutually exclusive. On contrary, we shall show that these desirable attributes, may even be mutually conditional.
(ii) We propose enabling strategies which lead to a clear-cut distinction between the *scientific computing* and *engineering* challenges in EM computations.
(iii) Focusing on the Boundary Element Method we shall exemplify the underlying ideas in the proposed method: We will explain the design of an engine which computes certain "Universal Functions" which are as fundamental as the Green's functions associated with a particular class of PDEs. However, in contrast to the Green's functions, these functions are, by way of construction, devoid of singularities, and possess many desirable properties such as the increased smoothness. The construction of Universal Functions will enable us to clearly demonstrate the aforementioned separability between the scientific computing and *engineering* tasks.

We refer to our analysis technique as the Fast-MoM, which is a *data recycling* concept. Since its inception in 1995, we have applied the Fast-MoM to the 2D solution of a variety of problems in acoustics and electromagnetics, and have demonstrated its superiority to the standard MoM. More recently we have addressed 3D problems.

The presentation starts with a review of the theory underlying the Fast-MoM, and briefly discusses a selected number of previous results. Assuming an N-manifold (N=2,3), and adopting a suitable local coordinate system, we then proceed as follows:

(1) The Boundary Element Method formulations are based on weak-, strong-, or hyper singular surface integral equations which involve Green's functions and/or their spacial derivatives as kernels. This fact justifies a thorough and systematic investigation of typical Green's functions. We present four different techniques for the construction of Green's functions, and discuss the resulting equivalent representations for the Green's functions, their spacial derivatives, moments, and their near- and far-field asymptotic behavior. The construction methods are, respectively, based on

- (a) the operator inversion,
- (b) the concept of Galerkin vector (a classical result revisited),
- (c) the spectral decomposition in N-dimensions, and
- (d) the spectral decomposition in (N-1)-dimensions.

Utilizing the (distributional) equivalence between various Green's functions' representations we "factorize" the Euclidean distance between the source- and observation points into terms which, respectively, only depend on the variables along the coordinate axis. Thereby, the N-dimensional decomposition leads to symmetric-, and the (N-1)-dimensional representation to asymmetric formulas with respect to the coordinates. The (N-1)- and N formulations permit the calculation of their respective Universal Functions in (N-1) and N-dimensions. In practice, however, the (N-1)-formulations are preferred due to the obvious reduced dimensionality.

(2) Using the concept of tangent spaces, and building upon our earlier works on the diagonalization of Maxwell's equations, we present a recipe for the construction of Universal Functions, provide a physical interpretation, and discuss our recent insight in regularizing ill-conditions integrals, which underlie these functions.

(3) Finally, we show that the presented concepts apply equally well to the Green's functions, which can be obtained in closed-form, as well as to the Green's functions, which are only available numerically.