Application of B-Spline and Dual B-Spline Wavelets to EM Scattering Problems

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In this presentation we discuss the construction and properties of B-splines, B-spline wavelets, dual B-spline wavelets, and their application to the EM scattering problems.

Affine functions can be conveniently manipulated by digital computers: In particular cases these transforms reduce to the translation, dilation (scaling), and rotation operations. Therefore, it would be beneficial to construct analyzing (approximating) functions which can be deduced from affine transformations. In this contribtion we address this need by systematically reviewing the well-established key ideas, and reporting our own most recent research results.

The presentation is organized as follows:
(1) Harmonic and non-harmonic analysis, the Littlewood-Paley decomposition in real harmonic analysis, coherent states expansions (inspired by constructive quantum field theory applications), pyramidal algorithms and multiscale filtering, and subband coding schemes will be briefly reviewed in the context of electromagnetic field analysis.
(2) A group theoretical interpretation of Gabor transform and coherent states, as far as relevant to the EM modeling, will be presented.
(3) Compactly supported wavelets including the B-spline wavelets, and dual B-spline wavelets will be discussed and an efficient implementation scheme will be proposed to tackle boundary value problems in electromagnetics. Special attention will be paid to the development of simple recipes for the construction of splines and the associated spline wavelets, and their representation in terms of simple “networks.” Thereby, we merely utilize the symbolic tools available in the linear algebra packages.
(4) The constructed scaling functions and wavelets in terms of the proposed matrices will be employed in solving Fredholm integral equations which arise in EM scattering problems. Metallic or dielectric scatterers with rather complex geometry, involving sharp edges, corners, and cusps will be considered. Where possible the numerical results will be compared with available data.
(5) The interaction elements of the involved “impedance matrices” contain Hankel functions of the second kind and zero order or their derivatives as kernels. A simple recipe for the calculation of the involved convolution integrals in real-space will be presented. The proposed technique solely requires sampled values of the kernel functions from which characteristic matrices are constructed. We diagonalize the resulting symmetric matrices, investigate the associated eigenpairs and study the spectral distribution of the eigenvalues.
(6) We discuss the thresholding process and its impact on the sparsity of the compressed impedance matrices and the accuracy of the results, by presenting a large number of examples.
(7) We conclude with comments on our ongoing work on wavelets as unconditional bases for the spaces Lp ($1<p$ but bounded), the trigonometric wavelets, and their application to the EM modeling.