# REPORT FOR THE SESSION ON STRONGLY INTERACTING MANY-PARTICLE SYSTEMS

IMS, SINGAPORE, SEPTEMBER 1 –21, 2008

## 1. Topics

The session was planned by Heinz Siedentop and Volker Bach as a series of introductory talks each followed by one or more sessions on open problems in the field. These served as starting point for the following discussions and work on the problems.

Generally speaking the program focussed on two major topics, namely:

- The mathematical theory of large atoms, molecules, and solids.
- The mathematical description of the radiation field and its interaction with matter.

These two topics were reviewed at the beginning of the session by Volker Bach and Heinz Siedentop.

The actually kick off discussion and work was generated through several series of seminars and open problem sessions of the discussions leaders. These were

(1) The asymptotic behavior of the ground state energy of large Coulomb systems (Heinz Siedentop, Thomas Østergaard Sørensen)
(2) Mean-Field model of relativistic Coulomb systems (Mathieu Lewin)
(3) Non-relativistic quantum electro-dynamics (Volker Bach, Jean-Marie Barbaroux)
(4) Exactly solvable models and strongly correlated two-dimensional systems (Edwin Langmann)
(5) The time-dependent Schrödinger equation: fundamental solutions

## 2. State of the discussions and work

The discussions and the detailed work on the above topics is as follows:

(1) Asymptotic behavior of the ground state energy of large Coulomb systems: after the introductory talks the discussion focused on

the description of heavy atoms and molecules where relativistic effects have to be taken into account. Starting from the Chandrasekhar operator more sophisticated models of relativistic matter were discussed. It was worked out that these models differ in their leading correction to the ground state energy (Scott correction) which is due to the innermost core electrons of atoms. We conjecture that for all reasonable pair Hamiltonians the difference to the non-relativistic Scott correction is give by the sum over the eigenvalue differences between the $n$-th eigenvalue of the bare no-pair operator and the $n$-th eigenvalue of the bare Schrödinger operator. This allows for direct comparison with experiment.

(2) Effective nonlinear models in relativistic no-photon QED. After a review session by Lewin, the participants have discussed on two nonlinear models of relativistic quantum mechanics: a Mittleman-type approach (of Bach, Barbaroux, Farkas, Helffer, and Siedentop) and the Bogoliubov-Dirac-Fock (BDF) model (of Hainzl, Lewin, Séré, Solovej, and Gravejat). The link between the works done by the two teams was made clearer in a long discussion. It was in particular mentioned that the BDF model can be expressed in a kind of generalized Mittleman formalism, and the approximations leading to the original Mittleman model were clearly identified.

Several open problems have been quickly discussed, like the possibility that the true no-photon QED vacuum is indeed highly correlated. Also the problem of renormalization was addressed. Although in a recent paper Gravejat, Lewin, and Séré have been able to define mathematically charge renormalization for a simplified model, the problem to give a rigorous meaning to mass and charge renormalization in no-photon QED seems a formidable task.

In another discussion session, some numerical issues specific to relativistic quantum mechanics were reviewed. The nonlinearity and the absence of a clear definition of the ground state are important problems for numerical methods aiming at describing systems with the Dirac operator. Another crucial issue is that of spectral pollution. It was in particular mentioned that there is at present no clear efficient method allowing to describe QED effects numerically which are free of spurious (unphysical) roots.

(3) Quantitative estimates on the ground state energy in non-relativistic quantum electro-dynamics (NRQED)

The estimate of the binding energy for Pauli-Fierz models in NRQED is related to the so-called "enhanced binding" and "increase of the binding energy" problems. In several rigorous results on the quantitative estimates of the binding energy, there was some intuition that in sufficient high order in the fine structure coupling constant $\alpha$, a term of the form $\alpha^n(\log \alpha)$ should occur, leading thus to non-analytic dependence of the binding energy in $\alpha$. This result was proved recently in the case of a spin-less electron coupled to the quantized radiation field.

Several discussions related to this model have been addressed in this meeting:

- Does this result hold true in the case of an electron with spin? It is expected that the spin of the electron should not change qualitatively the results. However, it occurs that one of the main ingredients, namely the photon soft bound, is in that case weaker, and thus induces a technical difficulty.
- What survives from these estimates if we consider the case of relativistic electron(s)? In that case, the lack of rigorous basic results available (existence of ground state for one or several electrons in the field of a nucleus, existence of a ground state for free electrons at total momentum zero, ...) renders every quantitative estimates a very difficult task to address. The problem is, however, highly interesting.
- In the limit of large $Z$, and, say, for $N = Z$ electrons, is it possible, *after a mass renormalization*, to prove that an first order in $Z^{\frac{7}{3}}$, the binding energy coincides with the Thomas-Fermi energy? Discussions in this meeting put in evidence that this is an ambitious problem to solve, that requires several non trivial, but interesting steps (see more details in other discussions on Scott correction):
  - For $N = 1$ electron, control the binding energy at sufficiently high order in $\alpha$, with exact estimates of the dependence in the ultra-violet cutoff $\Lambda$. Computation of the renormalized mass $m$, in terms of $\alpha$, $\Lambda$ and the physical mass $M$.
  - In NRQED of $N$ electrons, with the renormalized mass $m$ computed as above, estimate the binding energy in terms of the nuclear charge $Z$ (e.g. in the neutral case $N = Z$).

## 3. Tutorials

For the benefit of persons interested in getting into the field, in particular graduate students, Volker Bach gave a series of tutorial lectures. In these lectures he introduced the basic analytical tools on the one hand side. On the other hand he applied these to the most fundamental problems in non-relativistic quantum mechanics and showed how to prove – among other things – the “Stability of Matter”. The tutorials will be written up and will also serve faculty as material for introducing future students into the matter.

## 4. Interaction with Section 3

A particular fruitful aspect of the overall organization of the program was the two-weeks overlap of Sections 3 and 4. In particular, the mathematical questions on renormalization and the corresponding methods turned out to be of essential interest for the work in both sessions. In fact participants of our session and – vice versa – participants of Session 2 did not only take part in the seminars of the other session. In fact the input of Jan Derezinski who introduced the participants in an extra presentation into a general way of non-perturbative renormalization was also of utmost interest for us.

## 5. Feedback

We wish to conclude with a statement by Edwin Langmann (Royal Institute of Technology, Stockholm):

*During the last 50 years or so the theoretical physics community has accumulated much knowledge about quantum theory of large systems and, in particular, quantum field theory. However, much of this knowledge has not yet reached a stage which is satisfactory from a mathematical point of view, and there is a need for research on this topic which is at the interface of theoretical physics and mathematics which should lead to results which are useful to both disciplines: new mathematics will be discovered which then should provide new and more powerful tools in theoretical physics. The meeting in Singapore has assembled a group of mathematicians and theoretical physicists working in this fields.*

*The two activities going in parallel while I was visiting (open quantum systems and mathematical aspects of large atoms) complimented each other in a very good way: one important issue in the latter activity is to study the effect of the photons on atoms and, in particular, issues of renormalization needed in this context. During the meeting I became more aware of that this problem is closely related to open* *quantum systems (since the issue is to integrate out part of the degrees of freedom and determine the effect of these on the remaining smaller subsystem), even though the emphasis is somewhat different. In general, it was very interesting for me to have a chance to discuss with colleagues having a somewhat different emphasis in their research than me but working on very similar project.*

*I found this meeting very rewarding: I learned a lot and found it very inspiring for my research.*