Report

Workshop on Mathematical Horizons of Quantum Physics.
Session 2: Operator Algebras and Quantum Information Theory.

At the IMS, Singapore, August 11-31, 2008.
Organizers: Burkhard Kümmerer, Hans Maassen.

1 Outline

Leading idea of the Workshop (Sessions 1, 2, 3, and 4) is, rather than to offer a program of talks and lectures about work already done, to actually perform some research, and hence to make progress towards better understanding and even solution of important mathematical problems related to quantum physics.

Work in our Session on Quantum Information has been divided from the outset into three themes:

- **Stability of Quantum Algorithms.** Discussion leader: Mark Fannes.
- **Additivity of Channel Capacity.** Discussion leader: Alexander Holevo.
- **Entanglement.** Discussion leaders: Reinhard Werner and Mary-Beth Ruskai.

On each of the themes a tutorial was provided by an expert in the field (cf. Section 2), and groups of participants were formed around these themes. Then a "kick-off address" (cf. Section 3) was delivered by the leader of each group, meant to set the discussion in motion. On a daily basis the groups met at a fixed time, and between these times the participants were working and discussing, making use of the facilities of the IMS: the offices, the coffee room, the seminar room, the library, and the auditorium.

At the beginning a prize was written out to the nicest result obtained in collaboration during the meeting. It was awarded at the final meeting to four young researchers.

2 Tutorials

For the benefit of the graduate students, and in order to settle upon a common language and notation, tutorials were given on each of the three themes, plus an introduction to the central concept of quantum entropy.

- **Quantum entropy**,

  Mary Beth Ruskai, (Tufts University, USA), August 11.

- **Quantum memories as open systems,**

  Robert Alicki, (University of Gdańsk, Poland), August 12.

- **Entropic characteristics of quantum channels and the additivity problem,**

  Alexander Holevo, (Steklov Mathematical Institute, Moscow, Russia), August 12.

- **Entanglement generation in open quantum system,**

  Fabio Benatti, (University of Trieste, Italy), August 20.

3 Kick-off Adresses

The discussions in each of the groups was started by starting lectures or "kick-off addresses". They were held all on the same day, Wednesday August 13:

- **Stability of quantum algorithms,**

  Mark Fannes, (Katholieke Universiteit Leuven, Belgium).

- **Gaussian optimizers,**

  Alexander Holevo, (Steklov Mathematical Institute, Moscow, Russia),

- **Multipartite entanglement,**

  Reinhard Werner, (Technical University of Braunschweig, Germany),

4 Lectures

Two public lectures were given by key participants of our section:

- **Knot or not Knot?**

  Burkhard Kümmerer, (Technical University of Darmstadt), Germany; Wednesday, August 13.

- **Are Quantum Computers The Next Generation Of Supercomputers?**

  Reinhard Werner, (Technical University of Braunschweig), Germany; Wednesday, August 27.

At the Mathematics Department of the NUS a talk was given on the history of contacts between mathematics and physics, in particular Tomita-Takesaki theory:

- **Some Contact Points of Mathematics and Physics,**
  Huzihiro Araki, (Kyoto University, Japan), Wednesday August27.

In preparation for the latter, Burkhard Kümmerer gave an informal introduction to Tomita-Takesaki theory to the participants of our session on Monday August 25.

5 Results on the themes

• Stability of Quantum Algorithms.

1. For the question of stability of qubit storage, models are relevant of the coupling of two-level systems to the environment. One of these is the model proposed by Lebowitz and Pastur, where the coupling is described by a large random matrix. The discussion leader Mark Fannes proposed to treat this model more directly using Voiculescu's free probability calculus. During the workshop it was shown that indeed this translation is feasible. It is expected that the results of Lebowitz and Pastur can thus be obtained in a simpler way.

2. A second model of qubit storage is the Kitaev model. This is an N-spin model, where at time zero two logical qubits are encoded in a very delocalized way, which is a prerequisite for stability. Work already in progress by Paweł Horodecki, Robert Alicki, and Mark Fannes, was finished during the workshop, showing that increasing the size of system does not improve the thermal stability of the qubits in dimension 2.

3. The same technique was used in the opposite direction: a ground state gap of the system that closes with the system size proves stabilty of the encoded information. This is now believed to occur in dimension 4, on which work is forthcoming.

• Additivity of Channel Capacity. At the start of the meeting the discussion leader, Alexander Holevo, formulated several connected open problems related to additivity of minimal entropy and channel capacity.

1. The question of additivity of minimal output entropy of quantum channels, which is equivalent to the additivity of the Holevo capacity of such channels. On this open problem a partial result has been obtained in the course of session 2 by Thomas Wellens, Geza Giedke, Markus Tiersch, and Kay Schwiegert. It says that, when the minimal total output entropy of a pair of channels is zero, each channel in itself has

zero minimal entropy. The result may not be very strong in itself, but the method used indicates a way to attack the full problem. This result was rewarded the prize of 'nicest result', mentioned above.

Andreas Winter gave a talk on his negative result on the additivity of Renyi entropy around the parameter value $p = 0$.

2. It is conjectured that, in the case of a pair of Gaussian channels, the state optimizing the joint capacity is Gaussian. During the session it was shown by Alexander Holevo that, for the case of gauge invariant Gaussian channels, the above conjecture follows from another conjecture, saying that its minimum output entropy is realized in a Gaussian state.

• **Entanglement.** For a number of years it has been hoped in the Quantum Information community that, for a state on a pair of quantum systems, the property of having a positive partial transpose catches the essential characteristic of a separable state: namely the property of *not* being entangled in a distillable way. The prevailing opinion now tends to turn this hope down, but the question is still open.

The question was formulated by Reinhard Werner in his kick-off address, and, by initiative of Mary-Beth Ruskai, it became the focus of one of the groups. During the session the question was reformulated into operator algebraic language, and laid down in a short internal report by Geza Giedke and Hans Maassen. It turns out to be equivalent to the positivity of a well-defined series of operators on algebras. Talks were given on possible approaches to an answer by Zongwen Yu, Alexandra Liguori, Elisabeth Rieper, and Markus Tiersch.

• **Other Contacts.** In a visit to the staff seminar at the Center for Quantum Technologies, we learnt to our satisfaction that the orthogonal measures conjecture, formulated some 35 years ago by Alexander Holevo, had been solved for qubits by Andreas Keil.

A recent publication of Berthold-Georg Englert, Dagomir Kaszlikowski, Leong Chuan Kwek, and Wei Hui Chee on n-path interferometry contained a conjecture concerning the entropic uncertainty relation, which was refuted by Hans Maassen during the session, at least for certain values of the number n, and was replaced by a new one by Englert. A collaboration has started to sort out the matter.

6 Participants' feedback.

At the end of the meeting participants were asked in what way they had profited from the workshop. All expressed great appreciation. We cite a few reactions:

I feel that the visit was rather successful: between the talks, I found enough time to think, discuss with colleagues (mainly Markus and Geza), and starting to actually work on concrete problems. Thomas Wellens (Freiburg, Germany)

During the course of the present session, and with the help of fruitful discussions with other participants, I could make progress in the solution of the "Gaussian optimizers" conjecture. Alexander Holevo (Moskow, Russia)

All in all, this is the most excellent workshop that I have participated in. Zongwen Yu (Beijing, China)

This workshop gave me a great opportunity to meet with operator algebras in Quantum Information, ..., I am planning to study random matrices to understand the decoherence effects in simple models. Zeynep Nilhan Gurkan (Izmir, Turkey)

After this motivation we are thinking how to transfer our research direction to the fundamental NPT problem. Li Dafa (Beijing, China)

So I feel my time here in Singapore has been very well spent and I'm very thankful to the organizers for having given me the possibility to participate. Alexandra M. Liguori (Trieste, Italy)