Quantum Memories as Open Systems

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Robert Alicki

University of Gdańsk

Quantum Memory as a Challenge

- Scalable Quantum Memory Unit

  A scalable with $N \simeq 10, 100, 1000, ..$ quantum system composed of N physical qubits which supports a single (encoded) qubit. For achievable external conditions (low enough temperature, high vacuum, screening ,..., etc.) the life-time of encoded qubit observables increases like $\sim e^{\alpha N}$.

- The existence of Quantum Memory contradicts

- Bohr Correspondence Principle:

  Classical physics and quantum physics give the same answer when the systems become large.

- For large systems the experimental data are consistent with classical probabilistic models.

- Are there fundamental obstacles to build Scalable Quantum Memory?

Physical vs. encoded qubits

- Physical qubit - a natural system described by 2-dim Hilbert space and algebra $\mathcal{M}_2$ of 2×2 matrices spanned by $I, \sigma^x, \sigma^y, \sigma^z$.
- Examples: spin 1/2 , "2-level atom", bistable systems, etc.
- Encoded qubit - a subalgebra $\mathcal{Q}$ of the algebra $\mathcal{A}$ of the total system spanned by the self-adjoint elements I, X, Y, Z satisfying $X^2 = Y^2 = Z^2 = I$, $XY = iZ$, etc.(or generated by X, Y ; $X^2 = Y^2 = I$, $XY + YX = 0$)
- Examples ($\mathcal{A} = \mathcal{M}_{2^N}$ – N-physical qubits):
- 1) localized physical qubit - $X = \sigma_1^x \otimes I_{[2,N]}$, $Y = \sigma_1^y \otimes I_{[2,N]}$
- 2) 1D-Ising model , $H_N = -J \sum_{j=1}^{N} \sigma_j^z \sigma_{j+1}^z$, $b_j = \sigma_j^z \sigma_{j+1}^z$- bond $(N+1 \equiv 1)$
- a) $X = \sigma_1^x \otimes \sigma_2^x \otimes \cdots \sigma_N^x$, $Y = \sigma_1^y \otimes \sigma_2^x \otimes \cdots \sigma_N^x$
- b) $X' = XF_x$, $Y' = YF_y$, $F_{x,y}^\dagger = F_{x,y}$, $F_{x,y}^2 = I$, $F_{x,y}$- function of bonds.
- Remark: 2b) - most general encoded qubit commuting with bonds.

Noise and Errors

- 1)Macroscopic (engineered) noise and errors - not considered here.
- 2) Microscopic (thermal) noise - fundamental, inescapable
- Collective heat bath

$$H_N^{int} = \sum_{\alpha=z,x} \Big(\sum_{j=1}^{N} \sigma_j^{\alpha} \Big) \otimes F^{\alpha}$$

- leads to decoherence-free subsystems.
- Private heat baths

$$H_N^{int} = \sum_{\alpha=z,x} \sum_{j=1}^{N} \sigma_j^{\alpha} \otimes F_j^{\alpha}$$

- generic, ergodic coupling, $\lim_{t\to\infty} \rho_N(t) = \rho_N^{eq}$ - thermal equilibrium.

A generic model of classical memory

- Metastable local minima of free energy separated by free energy barriers $\sim N$ with exponentially long life-times are used to encode classical information.
- Examples: Classical Ising models - encoding a single bit (magnetization sign)
- Mean-field Ising $(\sigma_j = \pm 1)$ $\quad H_N^{mf} = -\frac{J}{2N}\sum_{i,j=1}^{N}\sigma_i\sigma_j$
- 1D-Ising $\quad H_N^{1D} = -J\sum_{,j=1}^{N}\sigma_j\sigma_{j+1}$
- Energy difference between
$+ + + + + + + + + + + + + +$ and $+ + +\underbrace{- - - - -}_{k-times} + + + + + +$

$$\Delta E^{mf} = Jk + \frac{k^2}{2N}\ ,\ \Delta E^{1D} = 2J$$

- Mechanism of bit's protection against noise. Phase transition for mean-field but not for 1D.
- Does it work for a qubit ?

Dynamical approach

- Glauber dynamics for classical Ising models
  $s = \{\sigma_k, k = 1, 2, ..., N\}$– configuration of N spins,
  s^j –configuration s with $\sigma_j \to -\sigma_j$
- Markovian Master Equation

$$\frac{d}{dt} P_s = \gamma \sum_{j=1}^{N} \left(P_{s^j} - e^{-\frac{E_j(s)}{kT}} P_s \right)$$

  where $E_j(s) = H_N(s^j) - H_N(s)$, γ - relaxation rate
- Gibbs state

$$P_s^{eq} = Z^{-1} e^{-\frac{H_N(s)}{kT}}$$

  is invariant and relaxing, detailed balance holds.
- Standard description of classical metastability.

Quantum Master Equation

- $\rho(t)$ - reduced density matrix of N-qubit system , $H|s\rangle = E_s|s\rangle$
- Transition maps

$$S_j^\alpha(\omega) = \sum_{E_{s'} - E_s = \omega} |s'\rangle\langle s'|\sigma_j^\alpha|s\rangle\langle s|$$

$\alpha = z, x,\ j = 1, 2, ..., N,\ \{A, B\} \equiv AB + BA$

$$\frac{d}{dt}\rho = -i[H, \rho] + \gamma \sum_{\omega \geqslant 0} \sum_{j=1}^{N} \big(S_j^\alpha(\omega)\rho S_j^\alpha(\omega)^\dagger - \frac{1}{2}\{S_j^\alpha(\omega)^\dagger S_j^\alpha(\omega), \rho\}\big)$$

$$+\gamma \sum_{\omega \geqslant 0} \sum_{j=1}^{N} e^{-\frac{\omega}{kT}} \big(S_j^\alpha(\omega)^\dagger \rho S_j^\alpha(\omega) - \frac{1}{2}\{S_j^\alpha(\omega) S_j^\alpha(\omega)^\dagger, \rho\}\big)$$

- Both Master Eqs. can be derived from Hamiltonian models using weak coupling limit (Davies, CMP 1974)

Properties of Davies generators

- Schroedinger picture

$$\frac{d}{dt}\rho = -i[H,\rho] + \mathcal{L}\rho$$

1) Relaxation to equilibrium

$$\lim_{t\to\infty}\rho(t) = \rho^{eq} = e^{-\frac{H}{kT}}/\mathrm{Tr}\big(e^{-\frac{H}{kT}}\big)$$

2) Diagonal elements

$$P_s = \langle s|\rho|s\rangle$$

evolve independently according to the classical Master Equation.

- Heisenberg picture

$$\frac{d}{dt}A = i[H,A] + L\rho$$

$\mathrm{Tr}\big(\rho(t)A(0)\big) = \mathrm{Tr}\big(\rho(0)A(t)\big)$, $t \geqslant 0$.

- 1) Hamiltonian part $i[H,\cdot]$ commutes with the dissipative part L.
- Algebra of observables equipped with the scalar product

$$\langle A,B\rangle_{eq} = \mathrm{Tr}\big(\rho^{eq} A^{\dagger} B\big)$$

  is a Hilbert space $\mathcal{H}^{eq}$
- 2) L is self-adjoint on $\mathcal{H}^{eq}$, $i[H,\cdot]$ is skew-adjoint
- 3) Spectral decomposition

$$A(t) = \sum_{\mu} e^{(i\omega_\mu - \lambda_\mu)t} \langle X_\mu, A(0)\rangle_{eq} X_\mu \ , \ \lambda_\mu > 0$$

- Spectral properties determine whether the system can serve as a Quantum Memory.
- Analysis of 1D , 2D and 4D Kitaev models in terms of Davies generators - R.A. , M Fannes and M. Horodecki.

Summary

- Existence of scalable Quantum Memory (Quantum Computer as well) contradicts Bohr Correspondence Principle.
- There is no ultimate proof of existence or nonexistence of Quantum Memory.
- There are interesting candidates for QM (topological degrees of freedom , e.g. 4D Kitaev).
- Technique of Davies generators seems to be a natural mathematical framework for these problems.

References

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