A discontinuous Galerkin method for elliptic interface problems with application to electroporation

Grégory Guyomarc'h and Chang-Ock Lee Division of Applied Mathematics, KAIST, Daejeon, 305-701, Korea colee@amath.kaist.ac.kr

We present a discontinuous Galerkin (DG) method to solve elliptic interface problems for which discontinuities in the solution and in its normal derivatives are prescribed on an interface inside the domain. Standard ways to solve interface problems with finite element methods consist in enforcing the prescribed discontinuity of the solution in the finite element space. Here, we show that the DG method provides a natural framework to enforce both discontinuities weakly in the DG formulation provided that the triangulation of the domain is fitted to the interface. The resulting discretization leads to a symmetric system that can be efficiently solved with standard algorithms. The method is shown to be optimally convergent in the L^2 -norm and numerical experiments are presented to confirm this theoretical result. We apply our method to the numerical study of electroporation, a widely-used medical technique with applications to gene therapy and cancer treatment. Mathematical models of electroporation involve elliptic problems with dynamic interface conditions. We discretize such problems into a sequence of elliptic interface problems that can be solved by our method. We obtain numerical results that agree with known exact solutions.