Uniqueness Conditions for a Hyperbolic-elliptic Model in Phase Transitions and Its two-phase Solution

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In the literature, many people have used *one-dimensional stress theory* to study boundary-value and/or initial-value problems of phase-transforming materials. Here, we shall show that these *pure* one-dimensional theories may have some essential defects. More specifically, we reveal that for these materials, both from theoretical considerations and experimental observations, the radial deformation and traction-free boundary conditions play some essential roles. From the point of view of nonlinear waves, this implies that one needs to take into account the dispersion for the propagation of the phase boundary in a slender cylinder. Here, we shall use two approaches to establish the proper model equations. One is based on the Whitham's theory for nonlinear dispersive waves and another is to use a novel asymptotic and series expansions to directly derive the model equation from the three-dimensional field equations. The nonlinear dispersive equation obtained by us shows that the problem is a singular perturbation one. The model equations used in the literature are only the leading order equations valid in the outer regions. By using our model equation and matching its traveling wave solution to those in the outer regions, we obtain three relations for three unknowns aross the phase boundary, which provide the uniqueness conditions for solutions. For a particular form of the stress function, we also provide the explicit solution for the two-phase deformation.

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