Some Mathematical Problems in Fluid Dynamics

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The first part of this talk is concerned with some important properties of the spherically symmetric solutions of the compressible Navier-Stokes equations. It is well known that spherically symmetric solutions (ρ, u) satisfies the system as follows

$$(1) \quad \begin{cases} \rho_t + (\rho u)_x + m\rho u/x = 0 \\ (\rho u)_t + (\rho u^2 + P(\rho))_x + m\rho u^2/x = \nu u_{xx} + \nu m(u/x)_x, \end{cases}$$

where ρ and u denote the density and velocity. One of the main results is

Theorem 1 *Let* (ρ, u) *be a weak solution of* (1). *If* $\int_E \rho(x,0)dx > 0$ *for every open set* $E \subset (0, +\infty)$,*then* $\int_E \rho(x,t)dx > 0$ *for every open set* $E \subset \bar{E} \subset (\omega(t), +\infty)$ *and for every* $t \in [0, T]$,where $\omega(t) = \sup\left\{y > 0 : \int_0^y \rho(x,t)x^m dx = 0\right\}$.

The theorem implies that if vacuum does not appear at the initial time, then it does not appear forever. In addition, other results on the properties of solutions are obtained, one of which implies that if two vacuum regions do not overlap at the initial time, then it does not appear forever.

In the second part of the talk, we present the general filtration equation

$$u_t - \Delta\phi(u) = 0, \tag{2}$$

where ϕ is a smooth and strictly increasing function. One of the fundamental problems is that whether or under what conditions the weak solutions of (2) are continuous. Due to the strong degeneracy of (2), such problem is rather difficult comparing with the classical case and the case of few points of degeneracy. By means of an approach different from those usually applied in studding the same kind of problem and a quite lengthy derivation, we arrive at an answer to this problem. Namely, we have

Theorem 2 *Let $u_0(x)$ be a bounded and measurable function. Then the bounded weak solution of (2) with initial data u_0 is continuous for $t > 0$,if $\Delta\phi(u_0) \geq 0$ (or $\Delta\phi(u_0) \leq 0$) in the sense of distribution.*