

On incompressible Navier-Stokes dynamics: a new approach for analysis and computation

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Abstract

The pressure term has always created difficulties in treating the Navier-Stokes equations of incompressible flow, reflected in the lack of a useful evolution equation or boundary conditions to determine it. In joint work with Bob Pego and Jie Liu, we show that in bounded domains with no-slip boundary conditions, the Navier-Stokes pressure can be determined in a such way that it is strictly dominated by viscosity. As a consequence, in a general domain with no-slip boundary conditions, we can treat the Navier-Stokes equations as a perturbed vector diffusion equation instead of as a perturbed Stokes system. We illustrate the advantages of this view by providing simple proofs of (i) the stability of a difference scheme that is implicit only in viscosity and explicit in both pressure and convection terms, requiring no solutions of stationary Stokes systems or inf-sup conditions, and (ii) existence and uniqueness of strong solutions based on the difference scheme.

A preprint is available at <http://arxiv.org/abs/math.AP/0502549>