

On the interaction of delta-shocks and contact vacuum states

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Abstract

We study the pressureless gas equations, with piecewise constant initial data. In the immediate solution, δ -shocks and contact vacuum states arise and even meet (interact) eventually. A solution beyond the “interaction” is constructed. It shows that the δ -shock will continue with the velocity it attained instantaneously before the time of interaction, and similarly, the contact vacuum state will move past the δ -shock with a velocity value prior to the interaction. We call this the “no-effect-from-interaction” solution.

We prove that this solution satisfies a family of convex entropies (in the Lax’s sense). Next, we construct an infinitely large family of weak solutions to the “interaction”. Suppose further that any of these solutions satisfy a convex entropy, it is necessary and sufficient that these solutions reduce to only the “no-effect-from-interaction” solution. In [?], Bouchut constructed another entropy satisfying solution. As with other previous papers, it is obvious that it will not be sufficient that a “correct” solution satisfies a convex entropy, in a non-strictly hyperbolic conservation laws system.