# Bubbling Accumulations For Nonlinear Elliptic Equations with Critical Nonlinearity

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**Abstract:** We consider the following nonlinear elliptic equation

$$(1) \quad \Delta u - \mu u + u^{\frac{N+2}{N-2}} = 0 \text{ in } \Omega, \quad u > 0 \text{ in } \Omega \text{ and } \frac{\partial u}{\partial \nu} = 0 \text{ on } \partial\Omega,$$

where $\Omega$ is a bounded and smooth domain in $R^N, N \geq 7$ and $\mu > 0$. (Joint work with C.-S. Lin.) We show that at a positive nondegenerate local minimum point $Q_0$ of the mean curvature, (we may assume that $Q_0 = 0$), for any fixed integer $K \geq 2$, there exists a $\mu_K > 0$ such that for $\mu > \mu_K$, the above problem has $K-$ *bubble* solution $u_\mu$ concentrating at the same point $Q_0$. More precisely, we show that $u_\mu$ has $K$ local maximum points $Q_1^\mu, ..., Q_K^\mu \in \partial\Omega$ with the property that $u_\mu(Q_j^\mu) \sim \mu^{\frac{2}{N-2}}, Q_j^\mu \to Q_0, j = 1, ..., K$, and $\mu^{\frac{3-N}{N}}(Q_1^\mu, ..., Q_K^\mu)$ approach an optimal configuration of the following functional

($*$) *Find out the optimal configuration that minimizes the following functional:*

$$R[Q_1, ..., Q_K] = c_1 \sum_{i=1}^{K} \varphi(Q_j) + c_2 \sum_{i \neq j} \frac{1}{|Q_i - Q_j|^{N-2}},$$

where $c_1, c_2 > 0$ are two generic constants and $\varphi(Q) = Q^T \mathbf{G} Q$ with $\mathbf{G} = (\nabla_{ij} H(Q_0))$.