Connectivity of Random Geometric Graphs

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Let $\mathcal{P}$ be a Poisson process of intensity one in a square $S_n$ of area $n$. We construct a random geometric graph $G_{n,k}$ by joining each point of $\mathcal{P}$ to its $k$ nearest neighbors. Recently, Xue and Kumar proved that if $k = 0.074 \log n$ then the probability that $G_{n,k}$ is connected tends to zero as $n \to \infty$, while if $k = 5.1774 \log n$ then the probability that $G_{n,k}$ is connected tends to one as $n \to \infty$. They conjectured that the threshold for connectivity is $k = \log n$. We improve these lower and upper bounds to $k = 0.3043 \log n$ and $k = 0.5139 \log n$ respectively, disproving this conjecture. We also establish lower and upper bounds of $k = 0.7209 \log n$ and $k = 0.9967 \log n$ for the directed version of the problem.