On the Signal-to-Interference-Ratio of CDMA Systems in Wireless Communications

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Abstract

Let $\{s_{ij} : i,j = 1,2,\dots\}$ consist of i.i.d. random variables in $\mathbb{C}$ with $\mathsf{E}s_{11} = 0$, $\mathsf{E}|s_{11}|^2 = 1$. For each positive integer N let $s_k = s_k(N) = (s_{1k}, s_{2k}, \dots, s_{Nk})^T$, $1 \le k \le K$, with $K = K(N)$ and $K/N \to c > 0$ as $N \to \infty$. Assume for fixed positive integer L, for each N and $k \le K$ $\alpha_k = (\alpha_k(1), \dots, \alpha_k(L))^T$ is random, independent of the s_{ij}, and the empirical distribution of $(\alpha_1, \dots, \alpha_K)$, with probability one converges weakly to a probability distribution H on $\mathbb{C}^L$. Let $\beta_k = \beta_k(N) = (\alpha_k(1)s_k^T, \dots, \alpha_k(L)s_k^T)^T$ and set $C = C(N) = (1/N)\sum_{k=2}^K \beta_k\beta_k^*$. Let $\sigma^2 > 0$ be arbitrary. Then with probability one $\mathrm{SIR}_1 \equiv \lim_{N\to\infty}(1/N)\beta_1^*(C + \sigma^2 I)^{-1}\beta_1 = \sum_{\ell,\ell'=1}^L \bar{\alpha}_1(\ell)\alpha_1(\ell')a_{\ell,\ell'}$ where $A = (a_{\ell,\ell'})$ is nonrandom, Hermitian positive definite, and is the unique matrix of such type satisfying $A = \left(c\,\mathsf{E}\frac{\alpha\alpha^*}{1+\alpha^* A\alpha} + \sigma^2 I_L\right)^{-1}$ where $\alpha \in \mathbb{C}^L$ has distribution H. The quantity SIR_1 is used as a model for the best signal-to-interference ratio for one user with respect to other $K-1$ users, for K large in a direct-sequence code-division multiple-access system in wireless communications. The result generalizes those previously derived but under more restricted assumptions, allowing for the analysis of user location with respect to the antennas in arbitrary settings (joint work with Zhidong Bai).