Semicircle Law for Hadamard Products

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In this paper, assuming $p/n \to 0$ as $n \to \infty$, we will prove the weak and strong convergence to the semicircle law of the empirical spectral distribution of the Hadamard product of a normalized sample covariance matrix and a sparsing matrix, which is of the form $A_p = \frac{1}{\sqrt{np}}(X_{m,n}X_{m,n}^* - \sigma^2 n I_m) \circ D_m$, where the matrices $X_{m,n}$ and D_m are independent and the entries of $X_{m,n}$ ($m \times n$) are independent, the matrix D_m ($m \times m$) is Hermitian with independent entries above and on the diagonal, p is the sum of the second moments of the row (and column) entries of D_m, and "$\circ$" denotes the Hadamard product of matrices.