Natural Exponential Families and Approximation Operators

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An exponential operator $S_{\lambda}(f,t)$ has the form $\int_{\mathbb{R}} W_{\lambda}(x,t) f(x) dx$ where f is a continuous function with compact support. It is assumed that

$$\partial_t \int_{\mathbb{R}} W_{\lambda}(x,t) f(x) dx = \int_{\mathbb{R}} W_{\lambda}(x,t) \frac{u-t}{p(t)} f(x) dx,$$

and mild assumptions on p. It turns out that t is the first moment. This concept was introduced in the 1970's and rediscovered in a statistical context in the 1980's. We summarize the developments in this area and mention some open problems.