Aspects of the Probability Theory of the MST and TSP

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This talk reviews two recent developments in the probability theory of the minimal spanning trees (or MSTs). First, it addresses a new exact formula for the expected value of the MST with random edge weights. Specifically, it notes that if G is a finite connected graph with Tutte polynomial $T(G; x, y)$, then for independent edge lengths that are uniformly distributed on $[0, 1]$, one has

$$E[L_{MST}(G)] = \int_0^1 \frac{(1-p)}{p} \frac{T_x\big(G; 1/p, 1/(1-p)\big)}{T\big(G; 1/p, 1/(1-p)\big)}\, dp, \qquad (1)$$

where $T_x(x, y)$ denotes the partial derivative of $T(x, y)$ with respect to x.

Second, the talk describes the probability theory of the PWIT, or Poisson Weighted Infinite Tree which was introduced by Aldous and which serves as universal limit for many natural probability problems in the theory of combinatorial optimization. The PWIT also provides a leading example of "objective method," the benefits and limitations of which are illustrated by a brief discussion of the limit theory of the MST, the assignment problem, and the random asymmetric TSP.