

Normal Approximation for Hierarchical Sequences

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Given $F : [a, b]^k \rightarrow [a, b]$ and X_0 with $P(X_0 \in [a, b]) = 1$, define the hierarchical sequence of random variables $\{X_n\}_{n \geq 0}$ by $X_{n+1} = F(X_{n,1}, \dots, X_{n,k})$, where $X_{n,i}$ are i.i.d. as X_n . Hierarchical sequences have been extensively used in the physics literature, for example, as a model for the conductivity of a random medium. Under an averaging and smoothness condition on non-trivial F a upper bound of the form $C\gamma^n$ for $0 < \gamma < 1$ is obtained on the Wasserstein distance between the standardized distribution of X_n and the normal. These bounds apply to hierarchical variables generated by certain weighted networks of which the resistor network is a special case. The results show how the weighted diamond lattice exhibits a full range of convergence rate behavior.