Stein's Method and
Edgeworth Expansions for Sums of Dependent Variables

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The paper concerns a development of Stein's method in a way that would allow to obtain asymptotic expansions for sums of r.v's with a relatively general dependency structure. The approach is based on the following general representation:

$$E[Wf(W)] = \sum_{m=0}^{r} (v_{m+1}/m!)E[f^{(m)}(W) + R,$$

where f is an r times differentiable function, $f^{(m)}$ is its m-th derivative, W is a r.v. with finite first $r+1$ moments, v_m is its m-th cumulant, and R is a remainder which may be small under suitable conditions. The possibility of a representation of such a type was first pointed in Barbour [1]. The main term above specifies the proximity to normality in terms of cumulants which are small under mild requirements, provided W is close to normal. Thus the main conditions to be imposed should concern the remainder. The essential difficulty lies in the fact that it is hardly possible to estimate R efficiently in terms of cumulants or some other characteristics of W itself as a non-decomposable r.v., so if W is a sum of r.v.'s, an efficient representation should include some characteristics of dependency.

The core of the class of dependency structures considered in the paper, is the local dependency, but in fact, the class is essentially wider. The most typical example is mixing on graphs, that is, when the parameter indexing the summands, which is usually thought of as a "time" or "space" parameter, has values which may be identified with vertices of a graph. If the graph is a usual integer valued lattice R^k with edges connecting only nearest vertices, we deal with the usual mixing scheme for random fields, and for $k = 1$ - with a process on a line. If the graph is arbitrary the scheme is more complicated. This is especially true when the graph is random, and its structure may depend on the values of summands. Using the representation above we come to Edgeworth expansions for the so called dependency-neighborhoods chain structures. The last results are generalizations of those in Rinott and Rotar [2].

REFERENCES.

1. Barbour, A. D.: Asymptotic expansions based on smooth functions in the central limit theorem, Probab. Th. Rel. Fields, 72, 289-303, (1986).
2. Yosef Rinott and Vladimir Rotar, On Edgeworth Expansions for Dependency-Neighborhoods Chain Structures and Stein's Method, Th. Rel. Fields, to appear, (2003).