L^1 Bounds in Normal Approximation

Larry Goldstein
University of Southern California

The zero bias distribution W^* of W, defined though the characterizing equation $EWf(W) = \sigma^2 Ef'(W^*)$ for all smooth functions f, exists for all W with mean zero and finite variance σ^2. For W and W^* defined on the same probability space, the L^1 distance between F, the distribution function of W with $EW = 0$ and $\text{Var}(W) = 1$, and the cumulative standard normal Φ, has the simple upper bound

$$||F - \Phi||_1 \leq 2E|W^* - W|.$$

This inequality is used to provide explicit L^1 bounds with moderate sized constants for independent sums, combinatorial central limit theorems, and projections of cone measure on the sphere $S(\ell_n^p)$.