Some general facts concerning asymptotic proximity of distributions, and some particular limit theorems.

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The talk concerns three topics.

1. First, we touch briefly on asymptotic expansions for the distribution of sums in the case of dependency-neighborhoods chain structures with strong mixing.

2. *The results on this and next topic were obtained jointly with Yurii A. Davydov.* We consider a general theory for convergence

$$P_n - Q_n \to 0 \text{ as } n \to \infty,$$

where P_n and Q_n are probability measures in a complete separable metric space. The main point is that the sequences $\{P_n\}$ and $\{Q_n\}$ are not assumed to be tight. We compare different possible definitions of the above convergence, and establish some general properties. The results obtained complement and, to a certain extent, develop the theory built in works of D'Aristotile, Diaconis, and Freedman [1], and Dudley [2, Chapter 11].

3. The third part concerns the invariance principle without the classical condition of asymptotic negligibility of individual terms. More precisely, let independent r.v.'s $\{\xi_{nj}\}$ and $\{\eta_{nj}\}$ be such that

$$E\{\xi_{nj}\} = E\{\eta_{nj}\} = 0,\ E\{\xi_{nj}^2\} = E\{\eta_{nj}^2\} = \sigma_{nj}^2,\ \sum_j \sigma_{nj}^2 = 1,$$

and let the r.v.'s $\{\eta_{nj}\}$ be normal. We set

$$S_{kn} = \sum_{j=1}^k \xi_{nj}, \quad Y_{kn} = \sum_{j=1}^k \eta_{nj}, \quad t_{kn} = \sum_{j=1}^k \sigma_{nj}^2.$$

Let $X_n(t)$ and $Y_n(t)$ be continuous piecewise linear random functions with vertices at (t_{kn}, S_{kn}) and (t_{kn}, Z_{kn}), respectively, and let P_n and Q_n be the respective distributions of the processes $X_n(t)$ and $Y_n(t)$ in $\mathbb{C}[0,1]$. The goal is to establish necessary and sufficient conditions for convergence of $P_n - Q_n$ to zero measure not involving the condition of the asymptotic negligibility of the r.v.'s $\{\xi_{nj}\}$ and $\{\eta_{nj}\}$.

References

[1] D'Aristotile, A., Diaconis, P., and Freedman, D. (1988), *On merging of probabilities*, Sankhyǎ: The Indian Journal of Statistics, v.58, Series A, Pt.3, pp. 363-380.

[2] Dudley, R. M., (2002). *Real Analysis and Probability*, 2nd edition, Cambridge University Press.