# Financial Contracts and the Theory of Debt 

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Outline of Part 1

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## 1 Introduction

## Question: How Should Firms Finance Operations?

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1. Direct Investment: Debt, Equity or Other
}
2. Financial Intermediation: Financial Intermediaries may Pareto dominate direct investment due to gains from
(a) Risk Diversification (credit or liquidity): Asset Transformation ${ }^{1}$
(b) Delegated Monitoring (return or effort): Banks Produce Information
(c) Offer Access to a Payments System: ${ }^{2}$ Exchange currency and guarantee the "finality" of payment.
(d) Enforcement Powers: Conferred by the legal system

Answer: In this lecture we will (mostly) focus on direct investment as an optimal contract problem. However, many of the issues that are important for understanding firm capital structure (i.e., debt versus equity) are important for understanding financial intermediation (e.g., asymmetric information, monitoring, enforcement, etc.)

[^1]
## 2 Finance in GE: The Arrow-Debreu Model

Arrow-Debreu Model: Neither money, intermediaries (banks), nor a determinant capital structure are necessary unless there is some "friction" in the economy. To see this, consider the following three problems for three two-period lived agents:

### 2.1 The Household

The household chooses consumption $\left\{C_{1}, C_{2}\right\}$ and an allocation of saving $S$ between bank deposits $D$ and private securities $B_{h}$, to maximize utility subject to budget constraints. Let
$w_{1}=$ the initial endowment of the consumption good
$\pi_{f}=$ profit of the firm (distributed to the consumer)
$\pi_{b}=$ profit of the bank (distributed to the consumer)
$r=$ interest paid on private securities
$r_{D}=$ interest paid on bank deposits
Problem $P_{h}$ : Choose $\left\{C_{1}, C_{2}, D, B_{h}\right\}$ to maximize

$$
u\left(C_{1}, C_{2}\right)
$$

subject to:

$$
\begin{gathered}
C_{1}+B_{h}+D=w_{1} \\
C_{2}=\pi_{f}+\pi_{b}+(1+r) B_{h}+\left(1+r_{D}\right) D
\end{gathered}
$$

### 2.2 The Firm

The firm chooses investment level $I$, which it acquires from bank loans $L$ or by issuing securities $B_{f}$ to maximize profit. Let $f(I)$ denote the firm's investment project.

Problem $P_{f}$ : Choose $\left\{I, B_{f}, L\right\}$ to maximize

$$
\begin{gathered}
\pi_{f}=f(I)-(1+r) B_{f}-\left(1+r_{L}\right) L \\
\text { subject to: } I=B_{f}+L
\end{gathered}
$$

### 2.3 The Bank

The bank chooses the supply of bank loans $L$, its demand for deposits $D$, and the amount of securities to issue $B_{b}$, to maximize profit.

Problem $P_{b}$ : Choose $\left\{D, B_{b}, L\right\}$ to maximize

$$
\begin{aligned}
& \pi_{b}=r_{L} L-r B_{b}-r_{D} D \\
& \text { subject to: } L=B_{b}+D
\end{aligned}
$$

### 2.4 General Equilibrium Results

When all agents optimize and markets clear (i.e., there is a competitive equilibrium (CE)), straightforward maximization of these problems implies:

1. $r=r_{L}=r_{D}$
2. Banks make zero profit
3. MM1: ${ }^{3}$ Households are indifferent about the composition of savings: banks versus securities
4. MM2: Firms are indifferent about the composition of borrowing: bank loans versus securities
5. MM3: The size and composition of banks' balance sheets have no impact on other agents.

Check: Show that $r=r_{L}=r_{D}$.
Point: In the Arrow-Debreu complete markets world, banks and money are redundant, and the firm's capital structure is irrelevant. We need a "friction" to generate a role for intermediaries, money, or different types of financial instruments (e.g., debt versus equity). Standard frictions are: Asymmetric information, transaction costs, indivisibilities (finance constraints), and non-convexities (fixed fees) cause frictions. Recently, enforcement has also received attention as an important friction.

[^2]
## 3 Literature on Asymmetric Information

The following papers show that debt is an optimal response to information problems when verification is costly:

Townsend [12] Costly State Verification (CSV) Model: Debt is an optimal response to asymmetric information:

- Finitely many entrepreneurs and investors
- Asymmetric information: All agents know $F(y)$ ex-ante but only the entrepreneur costlessly observes $y$ ex-post.
- Ex-ante CSV technology with commitment: Public announcement of the state if CSV occurs.

Result: Debt is optimal because it minimizes monitoring costs, but it is not ex post efficient.

Gale-Hellwig Model [3]: Study debt contracts with CSV.

- Representative investor and entrepreneur pair
- Asymmetric information
- Ex-post CSV technology: Private announcement if CSV occurs.

Result: Simple debt is optimal because it minimizes monitoring costs, but it is not ex post efficient. Underinvestment can occur.

Williamson CSV Model [15] and [14]: Debt is the optimal loan contract, but equilibrium credit rationing may occur. ${ }^{4}$

- Infinitely many entrepreneurs and investors
- Asymmetric information
- Finance constraint: need $m>1$ investors to finance a project
- Ex-post CSV technology: Private announcement of the state with costs that are independent of $m$.

[^3]Result: A bank that issues debt to firms is optimal. This arrangement minimizes monitoring costs because the expected costs of monitoring the bank $\rightarrow 0$ as the size of the bank $m \rightarrow \infty$ when the costs are independent of $m .{ }^{5}$

Krasa-Villamil [6] Costly Enforcement model: Debt is optimal in the multistage game that underlies the CSV model when commitment to initial decisions is limited. The reason is that debt is "informationally minimal" and hence it reduces the propensity of agents to renegotiate the contract ex post.

- Multi-stage game with incomplete information
- Costly enforcement is a choice variable
- There is limited commitment to initial decisions and hence agents might wish to renegotiate

Result: Debt is optimal when commitment to the initial contract is limited, even when stochastic monitoring is possible.

The Costly Enforcement model solves the "ex-post inefficiency problem" in the CSV model.

## 4 The Lender-Borrower Relationship: Debt

Consider the Lender-Borrower relationship (cf., Freixas and Rochet [2], chapter 4). Debt contracts are common, and they tend to be less complex than theory predicts. ${ }^{6}$ This occurs because a complete contingent contract would need to specify in every state of nature and at every date:

[^4]1. The amount of repayment or additional loan.
2. The interest rate on remaining debt.
3. Any adjustments in collateral required by the lender.
4. The actions to be taken by the borrower (investments) or the investor (whether to enforce).

In practice, debt contracts are often simple: repayment obligations and collateral $(1,2,3)$ are specified. "Other actions" are left to the borrower unless default occurs because the contract is non-contingent when the firm is solvent. Similarly, in early models renegotiation (i.e., the ability to "settle out of court" and whether to enforce were ignored.

### 4.1 Why Risk Sharing is Not Enough: Symmetric Info

It is useful to begin by showing that risk aversion is not enough to justify fixed payment (debt) contracts. Consider an economy with:

- One good
- Two dates: $t=0,1$

At $t=0$ the borrower can invest loan $L$ in a technology that produces at $t=1$ the random return

$$
\tilde{y}=g(L),
$$

For simplicity, let $\tilde{y}=\left(y_{1}, y_{2}\right)$ (there is a high and low state only).
Assume the following:
A1: The borrower has no resources and borrows $L$ from the lender.
A2: Both agents consume only at $t=1$.
A3: Preferences are given by the $C^{2}$, concave and strictly increasing Von-Neumann utility functions $u_{L}$ for the lender and $u_{B}$ for the borrower.
A4: Symmetric Info: $\tilde{y}$ is observable by both agents ex post. The random return is not known by either agent ex ante.

A5: There is costless and perfect enforcement of contracts ex-post.

Under assumptions $A 4$ and $A 5$ agents can write a contract at $t=0$ which specifies how they will share $\tilde{y}$ at $t=1$. Once repayment $R(y)$ is specified as a function of realization $y$, the contract is completely determined:

- $R(y)$ : Investor payment
- $y-R(y)$ : Borrower return

A6: Limited Liability Constraint: $0 \leq R(y) \leq y$ for all $y \in \tilde{y} .^{7}$

### 4.2 Symmetric Information

The family of optimal debt contracts under symmetric information solve the following problem. Let $\bar{U}_{L}$ be the lender's reservation utility level.

Problem: Choose $R(\cdot)$ to maximize ${ }^{8}$

$$
E\left[u_{B}(\tilde{y}-R(\tilde{y}))\right]
$$

subject to:

$$
\begin{gather*}
E\left[u_{L}(R(\tilde{y}))\right] \geq \bar{U}_{L}  \tag{IR}\\
0 \leq R(y) \leq y \tag{LL}
\end{gather*}
$$

Since $u_{B}$ and $u_{L}$ are monotonic, the individual rationality constraint (IR) will always bind - the lender is driven down to reservation utility level $\bar{U}_{L}$. The "family of contracts" is indexed by this outside reservation value. ${ }^{9}$ Thus the roles of the borrower and lender are completely symmetric. The contract is determined completely by risk sharing and limited liability.

[^5]When the limited liability constraint does not bind, the solution implies that the marginal rates of substitution (MRS) across states for the two agents must be equalized (i.e., complete risk sharing): ${ }^{10}$

$$
M R S=\frac{u_{L}^{\prime}\left[R\left(y_{1}\right)\right]}{u_{L}^{\prime}\left[R\left(y_{2}\right)\right]}=\frac{u_{B}^{\prime}\left[y_{1}-R\left(y_{1}\right)\right]}{u_{B}^{\prime}\left[y_{2}-R\left(y_{2}\right)\right]}
$$

This means that the optimal contract is determined by optimal risk sharing. Equivalently, the ratio of the Marginal Utilities (MU) of the two agents is a constant $\mu$ for all $y \in \tilde{y}$, where $\mu$ depends on reservation value $\bar{U}_{L}$ :

$$
\frac{M U_{B}}{M U_{L}}=\frac{u_{B}^{\prime}[y-R(y)]}{u_{L}^{\prime}[R(y)]}=\mu
$$

Log the previous equation and differentiate it with respect to $y$ to get

$$
\frac{u_{B}^{\prime \prime}}{u_{B}^{\prime}}(y-R(y))\left(1-R^{\prime}(y)\right)-\frac{u_{L}^{\prime \prime}}{u_{L}^{\prime}}(R(y)) R^{\prime}(y)=0
$$

The index of absolute risk aversion for each agent is given by ${ }^{11}$

$$
I_{B}(x)=-\frac{u_{B}^{\prime \prime}(x)}{u_{B}^{\prime}(x)} \text { and } I_{L}(x)=-\frac{u_{L}^{\prime \prime}(x)}{u_{L}^{\prime}(x)}
$$

Result: When the limited liability constraint does not bind, the optimal debt contract satisfies ${ }^{12}$

$$
R^{\prime}(y)=\frac{I_{B}(y-R(y))}{I_{B}(y-R(y))+I_{L}(R(y))}
$$

This result says that the sensitivity of $R(y)$ to $y$ is high when the borrower is more risk averse than the lender (i.e., $\frac{I_{B}}{I_{L}}>1$ ). To see this, consider the two cases: ${ }^{13}$

[^6]

Figure 1: Simple Debt Contact

- When the borrower is risk neutral, $I_{B} \rightarrow 0$. Then $R^{\prime}(y)=0$. This corresponds to the flat part of the debt contract, where $R(y)=\bar{R}$ and $R^{\prime}(y)=0$. The key observation is that the payment is constant, thus it is not necessary to know the state. The lender only cares whether he/she was repaid, not about the underlying realization.
- When the borrower is risk averse, $I_{B}>0$. Then $R^{\prime}(y) \rightarrow 1$. This corresponds to the $45^{0}$ line in the debt contract, where $R(y)=y$ and $R^{\prime}(y)=1$. The point is that the payment is contingent, and a contingent payment requires all parties to know the state.

In summary, the consideration of risk (under symmetric information) leads to the following results.

Result 1: Risk aversion cannot explain a simple debt contract (SDC). Risk aversion tends to generate contracts that are highly state contingent (i.e., $R^{\prime}(y)=1$ ), and debt is not state contingent in "good times" (for $y \in B^{c}$ ).

Result 2: Banks are usually large and risk neutral. Thus, one might think that SDCs follow from risk neutrality because $R^{\prime}(y)$ is close to 0 . Although the typical bank contract involves a constant repayment $R(y)=\bar{R}$, when limited liability is introduced the repayment function becomes $R(y)=\min (y, \bar{R})$. See Figure 1. In the last section of this lecture we will see that simple debt is the optimal contract
when agents are risk neutral, but optimality is driven by minimizing monitoring costs - not risk neutrality. Risk neutrality alone would "miss" $y \in B$.

Result 3: Risk sharing alone cannot explain simple debt contracts. We will see that information asymmetry helps.

### 4.3 Brief Review of Risk Aversion

A classic reference for this material is Pratt [10]. Let
$x$ denote an amount of money
$F(x)$ denote a cdf that describes a money lottery
For any $x$,

- $F(x)$ is the probability that the realized $x_{i}$ is such that $x_{i} \leq x$.
- $\int u(x) d F(x)$ is the expected utility from non-negative amounts of money.

Definition. An agent is risk averse if for any lottery $F(\cdot)$, the degenerate lottery that yields $\int x d F(x)$ with certainty is at least as good as lottery $F(\cdot)$.

Definition. An agent is risk neutral if the agent is indifferent between the two lotteries.

Definition. An agent is strictly risk averse if the agent is indifferent only when the two lotteries are exactly the same. For all $F(\cdot)$,

$$
\int u(x) d F(x) \leq u\left(\int x d F(x)\right) .
$$

The inequality in the third definition is Jensen's inequality. This is the defining property of a concave function. Risk aversion is equivalent to concavity of $u(x)$.

Definition. Given a $C^{2}$ utility function, the Arrow-Pratt coefficient of absolute risk aversion at $x$ is

$$
I_{A}(x)=\frac{-u^{\prime \prime}(x)}{u^{\prime}(x)} .
$$

Risk neutrality is equivalent to linearity of $u(\cdot)$, with $u^{\prime \prime}(x)=0$ for all $x$. Thus, the degree of risk aversion is related to the curvature of the utility function; risk aversion increases as $u^{\prime \prime}(x)$ increases, where $u^{\prime \prime}(x)$ is a measure of the curvature of $u(\cdot)$. However, it is not invariant to positive linear transformations of $u(x)$. To make it invariant, Arrow and Pratt devised

$$
I_{A}(x)=\frac{-u^{\prime \prime}(x)}{u^{\prime}(x)} .
$$

The purpose of the sign change is to get a positive number for an increasing and concave function $u(\cdot)$. The Arrow-Pratt measure of absolute risk aversion fully characterizes behavior under uncertainty.

Example. Let $u(x)=-e^{-a x}$ for $a>0$. Then

$$
\begin{gathered}
u^{\prime}(x)=a e^{-a x} \\
u^{\prime \prime}(x)=-a^{2} e^{-a x}
\end{gathered}
$$

Therefore, for all $x$

$$
I_{A}(x)=\frac{a^{2} e^{-a x}}{a e^{-a x}}=a
$$

Finally, note the following standard properties:

- If $I_{A}\left(x, u_{2}\right) \geq I_{A}\left(x, u_{1}\right)$, then utility function $u_{2}(\cdot)$ is more risk averse that $u_{1}(\cdot)$.
- If $u(x)$ satisfies decreasing absolute risk aversion, then the agent takes more risk as he/she becomes wealthier.

Check: Suppose utilities are exponential,

$$
u_{i}(y)=-e^{-\rho_{i} y}, i=B, L
$$

1. Show that the absolute indexes of risk aversion are constant, $I_{i}(y)=\rho_{i}$, $i=, B, L$.
2. Show that the optimal repayment function is $R(y)=\alpha y+\beta$, with $\alpha=\frac{\rho_{B}}{\rho_{B}+\rho_{L}}$ The total repayment is $R(y)=\bar{R}+\alpha(y-\bar{R})$.

- $\bar{R}$ is a SDC.
- $\alpha(y-\bar{R})$ is equity participation.


### 4.4 Costly State Verification (CSV) Model

We now consider the CSV model which Townsend [12] used to rationalize debt. Asymmetric information and costly monitoring are the key frictions in the model. Suppose there are two periods $t=0,1$ with

- Symmetric info in the planning period $t=0$ : Both agents know the distribution $F(y)$.
- Asymmetric info in the consumption period $t=1$ : The borrower costlessly observes realization $\tilde{y}$ and the lender does not.
- In order to observe $\tilde{y}$, the lender must pay deadweight cost $c$.

| Agent | Pref. | Endowment | Techology | Information |
| :---: | :---: | :---: | :---: | :---: |
| Lender | $u\left(c_{1}(y)\right)$ | 1 unit input | no access | $F(y), \tilde{y}$ if CSV |
| Borrower | $u\left(c_{1}(y)\right)$ | no input | $[-m, \tilde{y}]$ | $F(y), \tilde{y}$ |

There are two types of agents: ${ }^{14}$

- $i=1, \ldots, I$ lenders
- $j=1, \ldots, J$ borrowers


## A.1: Agents are risk neutral.

A.2: $I=m J$. When $m>1$ there is a finance constraint and $J<I$ (there are more investors than projects). ${ }^{15}$
A.3: Project realizations $\tilde{y}$ are iid, and distribution $F(\cdot)$ has a continuously differentiable density $f(\cdot)$ with respect to the Lebesque measure, with $f(y)>0$ for all $\tilde{y} \in[0, T]$.
A.4: The CSV technology has the following properties

[^7]- Deterministic monitoring. ${ }^{16}$
- Revelation of $\tilde{y}$ may be public or private.
- Non-trivial monitoring costs: $c>0$.


## Notation:

$\tilde{y}=$ borrower's realization
$R(\tilde{y})=$ payment by the borrower to the lender
$F(y)=$ distribution of $y$ (known by all)
$c=$ lender's monitoring cost (occurs in "bankruptcy states")
$\bar{U}_{L}=$ lender's reservation level of utility
Because agents have asymmetric information, a key problem is to ensure that the borrower reports $\tilde{y}$ truthfully. A particular type of commonly observed contract, simple debt, solves this problem. We now summarize some important results from this literature (cf., Townsend [12], Gale and Hellwig [3], and Williamson [14]).

Definition. A contract between a lender and borrower is a pair $(R(\tilde{y}), B)$ where

- $0 \leq R(\tilde{y}) \leq \tilde{y}$ for every $\tilde{y} \leq R_{+}$is a payment function $R(\cdot)$ from the borrower to the lender;
- $B$ is a subset of $R_{+}$which determines the set of all realizations that are monitored (i.e., bankruptcy states).

Definition. $(R(\cdot), \bar{R})$ is a simple debt contract if

- $R(\tilde{y})=\tilde{y}$ for $\tilde{y} \in B$, and
- $R(\tilde{y})=\bar{R}$ if $\tilde{y} \in B^{c}$.

Without loss of generality, the universe of contracts can be restricted to the set of incentive-compatible contracts: the realization announced by each borrower is

[^8]

Figure 2: Simple Debt Contact
the true realization, $\tilde{y}=y$ (cf., Townsend [13]). Incentive compatibility is assured by restriction IC:

There exist $\bar{R} \in R_{+}$such that $B=\{y: R(y)<\bar{R}\}$.
The borrower never has an incentive to lie on the non-bankruptcy set, $B^{c}$, because the report is irrelevant. The lender only cares about payment $\bar{R}$, not the reported $\tilde{y}$ (since the payment is non-contingent). Thus we need only worry that on the bankruptcy set $y \in B$, the borrower is unable to pay $(R(y)<\bar{R})$.

IC contracts resemble simple debt because (cf. Williamson [14]:

- When verification occurs the payment to the lender is state contingent, and the borrower pays the entire $y$ for all outcomes below a cutoff level. The borrower gets the maximum penalty (i.e., zero consumption) in bankruptcy states, and this gives the borrower the incentive to tell the truth.
- When verification does not occur the payment to the lender is constant (i.e., the borrower pays a fixed amount $\bar{R}$ for all $y$ above the cutoff), where $B^{c}$ is the set of all realization where verification does not occur.

Without loss of generality we restrict contracts to the set of IC contracts. Henceforth, $\tilde{y}=y$. The primary investment problem between a borrower and a lender can now be stated. Recall that $c$ is the lender's cost of monitoring a borrower.

Problem. Choose an incentive-compatible contract $(R(\cdot), B)$ to:

$$
\begin{equation*}
\max \int_{0}^{T}[y-R(y)] d F(y) \tag{1}
\end{equation*}
$$

subject to:

$$
\begin{equation*}
(J / I) \int_{0}^{T} R(y) d F(y)-\int_{B} c d F(y) \geq(J / I) \bar{U}_{L} . \tag{2}
\end{equation*}
$$

The expected utility of the representative borrower is maximized subject to a constraint that the lender's expected return, net of monitoring costs (c), be at least as great as reservation level $\bar{U}_{L}$. The first term in the constraint is multiplied by $(J / I)$ to account for each individual lender's investment. ${ }^{17}$

Theorem SDC. Simple debt is the optimal contract. See Townsend [12], GaleHellwig [3] and Williamson [14].

Sketch of Proof: See the Figure below. Consider two incentive compatible contracts that give the same expected utility:

- $(\bar{R}, B)$ : Simple debt contract
- $(A, B)$ : An alternative contract that need not be simple debt

The intuition is as follows. Lowering face value $\bar{A}$ to $\bar{R}$ and lowering the investor's payoff in one of the bankruptcy states increases the entrepreneur's payoff. Therefore, SDC $R(\cdot)$, rather than arbitrary contract $A(\cdot)$, is optimal.

The strategy of the proof is as follows: Efficient IC debt contracts are obtained by minimizing the expected monitoring cost, for a fixed expected repayment. ${ }^{18}$

- Suppose by way of contradiction that there exists an alternative contract $A(\cdot)$, possibly not a SDC, with the same expected return as SDC $R(\cdot)$.
- Because $A(\cdot)$ has more default states, it must give the investor a higher face value $\bar{A}$ (to keep the expected return fixed).

[^9]

Figure 3: Optimal Simple Debt Contracts
Lowering face value $A$ to $R$ and lowering the lender's payoff in one of the bankruptcy states increases the borrower's payoff. Therefore, SDC $\bar{R}$, rather than arbitrary contract $A(\cdot)$, is optimal.

- This implies directly that the expected monitoring cost under $A(\cdot)$ is not minimal, a contradiction. Therefore the alternative (non-simple debt contract) is not optimal.

Given that simple debt is the optimal contract, the contract problem simplifies to finding the face value of the debt $\bar{R}$ that maximizes the borrower's expected payoff subject to the lender getting reservation value $\bar{U}_{L}$.

Check: Show that the SDC in the CSV model is not ex post efficient. That is, stochastic monitoring where verification of state $y_{i}$ occurs with probability $0 \leq$ $p_{i} \geq 1$ Pareto dominates deterministic monitoring which occurs with probability 1 or 0. See Townsend [12] or Moohkerjee and Png [9]. Stochastic contracts do not resemble SDCs because a stochastic contract is highly state-contingent (i.e., it does not have the fixed payoff that characterizes debt in most states).

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[^1]:    ${ }^{1}$ There are three main types of asset transformation: convenience of denomination, quality (risk) transformation and maturity transformation. We will focus on information and enforcement.
    ${ }^{2}$ Payment systems are networks that facilitate the transfer of funds between bank accounts and economic agents.

[^2]:    ${ }^{3}$ MM refers to the Modigliani-Miller Theorem or irrelevance results. Uncertainty does not change this result with complete markets (cf., Modigliani-Miller [8]).

[^3]:    ${ }^{4}$ In an important paper, Diamond [1] developed a delegated monitoring model of intermediation and used the law of large numbers to show that intermediation dominates direct investment.

[^4]:    ${ }^{5}$ When the bank's monitoring costs are independent of its size, this gives the bank a cost advantage (natural monitoring monopoly). Hence, dominance of intermediation over direct investment follows. Krasa-Villamil [4] and [5] considered delegated monitoring when the bank's portfolio has non-trivial risk and hence the bank may default (two-sided debt problem). Who monitors the monitor? This requires a stronger convergence result than the Law of Large Numbers; Krasa and Villamil use the Large Deviation Principle. Expanding bank size has two effects: (1) There is a gain from pooling risk (diversification), but (2) a loss from increased monitoring cost. This tradeoff leads to an optimal bank size. The Large Deviation Principle allows one to calculate the gain (in terms of better diversification) from adding an additional loan to the portfolio.
    ${ }^{6}$ See Smith and Villamil [11] for a theoretical justification of a particular debt instrument that involved explicit randomization.

[^5]:    ${ }^{7}$ Limited liability means that the court cannot seize the personal assets of the firm; the court can only seize returns from the business, for each realization $y$. In this simple case, this means state $y_{1}$ or $y_{2}$.
    ${ }^{8}$ The expectation is taken with respect to random variable $y$. Because there are only two states, the objective can be written $E\left[u_{L}(\tilde{y}-R(\tilde{y}))\right]=p_{1} u_{L}\left(y_{1}-R\left(y_{1}\right)\right)+p_{2} u_{L}\left(y_{2}-R\left(y_{2}\right)\right)$. We can write $E\left[u_{B}(\tilde{y}-R(\tilde{y}))\right]$ analogously. See Mas-Colell, Whinston (1995), chapter 19, or Ljungqvist and Sargent [7], chapter 7, for a review of General Equilibrium under uncertainty.
    ${ }^{9}$ We can also maximize the expected utility of the lender subject to an individual rationality constraint for the borrower and limited liability.

[^6]:    ${ }^{10}$ The ratio $\frac{p_{1}}{p_{2}}$, due to the expected utility maximization, appears on both sides of the equation below and hence cancels.
    ${ }^{11}$ See Mas-Colell, Whinston, Green (1995), chapter 6, for a discussion of choice under uncertainty. The basic idea is that agents whose preferences satisfy decreasing absolute risk aversion take more risk as they become wealthier. When an agent is risk neutral, then $u^{\prime \prime}(x)=0$. Thus, $I(x)=0$. The "more positive" $I(x)$ is, the more risk averse the agent is. See the next section of the notes for a brief review of the Arrow-Pratt measure of risk aversion.
    ${ }^{12}$ Take the second equation above, substitute in for $I_{B}$ and $I_{L}$, and solve it for $R^{\prime}(y)$. This is simple algebra.
    ${ }^{13}$ Contingent means $R(y)=y$ and non-contingent means $R(y)=\bar{R}$.

[^7]:    ${ }^{14}$ Sometimes it will be sufficient to assume that there are only two agents: $I=J=1$.
    ${ }^{15}$ This assumption is necessary for establishing results on financial intermediation, not the optimality of debt. Thus, we will often ignore this assumption.

[^8]:    ${ }^{16}$ This was a troublesome assumption that led to the "ex post inefficiency problem" in the CSV model. Early versions of the model restricted monitoring to occur with probability 1 or 0 . Stochastic (i.e., random) monitoring was ruled out, even though this can be efficient (e.g., tax authorities audit randomly). Further, since no one cheats in the model in equilibrium, it is inefficient to verify ex post because it wastes $c$. Krasa and Villamil [6] solve this problem.

[^9]:    ${ }^{17}$ Without loss of generality, assume that each lender invests all of his/her endowment in a single project. Credit markets are competitive since investors also have an outside alternative. Thus the return necessary to attract investors is reservation level $\bar{U}_{L}$, the return available on the alternative investment opportunity. Finally, in this section assume that there are only two agents, thus $I=J=$ 1 to simplify the model.
    ${ }^{18}$ Risk sharing is irrelevant in the argument because agents are risk neutral.

