Impact of wall functions in computational fluid mechanics

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Aim

Evaluate the impact of wall function modeling on

meshing,

simulation (CFL, condition of systems),

sensitivity evaluation,

and optimization

with fluid flows.

Mathematical context

General form

$$\frac{\partial W}{\partial t} + \nabla.(F(W) - N(W)) = S(W)$$

In weak formulations the following boundary integrals appear:

$$\int_\Gamma W(\vec{u}.\vec{n})d\sigma \qquad \int_\Gamma p\vec{n}d\sigma$$

$$\int_\Gamma p(\vec{u}.\vec{n})d\sigma \qquad \int_\Gamma (\mathbf{S}.\vec{n})d\sigma$$

$$\int_\Gamma (\vec{u}\mathbf{S})\vec{n}d\sigma \qquad \int_\Gamma (\chi + \chi_t)\frac{\partial T}{\partial n}d\sigma$$

$$\int_\Gamma (\mu + \mu_t)\frac{\partial k}{\partial n}d\sigma \qquad \int_\Gamma (\mu + c_\varepsilon \mu_t)\frac{\partial \varepsilon}{\partial n}d\sigma$$

Boundary conditions

with non-penetration boundary conditions, only remains

$$\int_\Gamma p\vec{n}d\sigma \qquad \int_\Gamma (\mathbf{S}.\vec{n})d\sigma$$

$$\int_\Gamma (\vec{u}\mathbf{S})\vec{n}d\sigma \qquad \int_\Gamma (\chi + \chi_t)\frac{\partial T}{\partial n}d\sigma$$

$$\int_\Gamma (\mu + \mu_t)\frac{\partial k}{\partial n}d\sigma \qquad \int_\Gamma (\mu + c_\varepsilon \mu_t)\frac{\partial \varepsilon}{\partial n}d\sigma$$

Dimension reduction

Aim: solve up to the wall on different meshes.

H1. Anisotropy normal to the wall

$$\partial_x, \partial_{x'}, \partial_t << \partial_y$$

$$\mathbf{S}.\vec{n} = (\mathbf{S}.\vec{n}.\vec{n})\vec{n} + (\mathbf{S}.\vec{n}.\vec{t}).\vec{t} + (\mathbf{S}.\vec{n}.\vec{t'}).\vec{t'}$$

H2. we neglect S_{nn} and $S_{nt'}$ ($\vec{t}//\vec{u}$)

$$\mathbf{S}.\vec{n} \sim (\mathbf{S}.\vec{n}.\vec{t}).\vec{t} = S_{nt}\vec{t}$$

u_τ defined as $\rho_w u_\tau |u_\tau| = S_{nt}$

Dimension reduction - 2

Specify $\int_\Gamma (\chi + \chi_t)\partial_y T$ from energy eq:

$$\partial_y(u(\mu + \mu_t)\partial_y u) + \partial_y((\chi + \chi_t)\partial_y T) = 0$$

integrate between $y = 0$ and $y = \delta$,

since $u\mid_0 = 0$

$$(\chi + \chi_t)\partial_y T\mid_\delta + u(\mu + \mu_t)\partial_y u\mid_\delta = \chi\partial_y T\mid_0$$

Rmq: on adiabatic walls this vanishes.

Dimension reduction - 3

Turbulence variables:

$$\partial_y((\mu + \mu_t)\partial_y k) = P_k(y) - \rho\varepsilon$$

where

$$P_k = (\rho_w u_\tau^2)^2/(\mu + \mu_t)$$

$$\mu_t^+ = \mu_t/\mu \sim l_\mu y^+$$

$$y^+ = \frac{\rho_w y u_\tau}{\mu_w}$$

same for ε or $= \frac{k^{3/2}}{l_\epsilon}$ (care compatibility !)

Closure: specify ρ_w, u_τ and $\chi\partial_y T \mid_0$ for isothermal walls.

Closure

Same anisotropy hypothesis implies:
$\rho_w u_\tau |u_\tau| = (\mu + \mu_t)\frac{\partial u}{\partial y}|_{y=\delta} = \mu\frac{\partial u}{\partial y}|_{y=0}$

integration for $0 \leq y \leq \delta$

$$\rho_w u_\tau |u_\tau| = \frac{u_\delta}{\int_0^\delta \frac{dy}{\mu+\mu_t}}$$

In the same way, $\chi\frac{\partial T}{\partial y} |_0 = \frac{T_\delta - T_w}{\int_0^\delta \frac{dy}{\chi+\chi_t}} + u_\delta \rho_w u_\tau |u_\tau|$

Above approach valid if the turbulence model is valid for the flow.

Care on mesh definition following δ.

Turbulence models weakness on compressible and thermal aspects:

Introduce local physics in wall functions.

Analytical wall functions

Push the integration above one step more to express

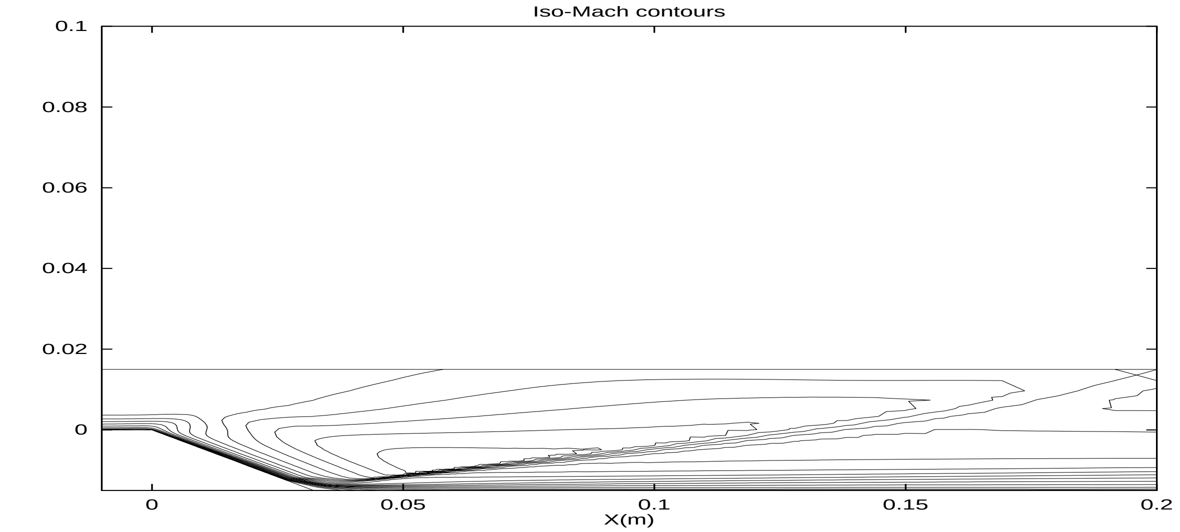
$u^+ = u/u_\tau(y^+)$ and $T^+ = T/T_w(y^+)$ for isothermal walls

+ physical information during this integration (not a unique way),

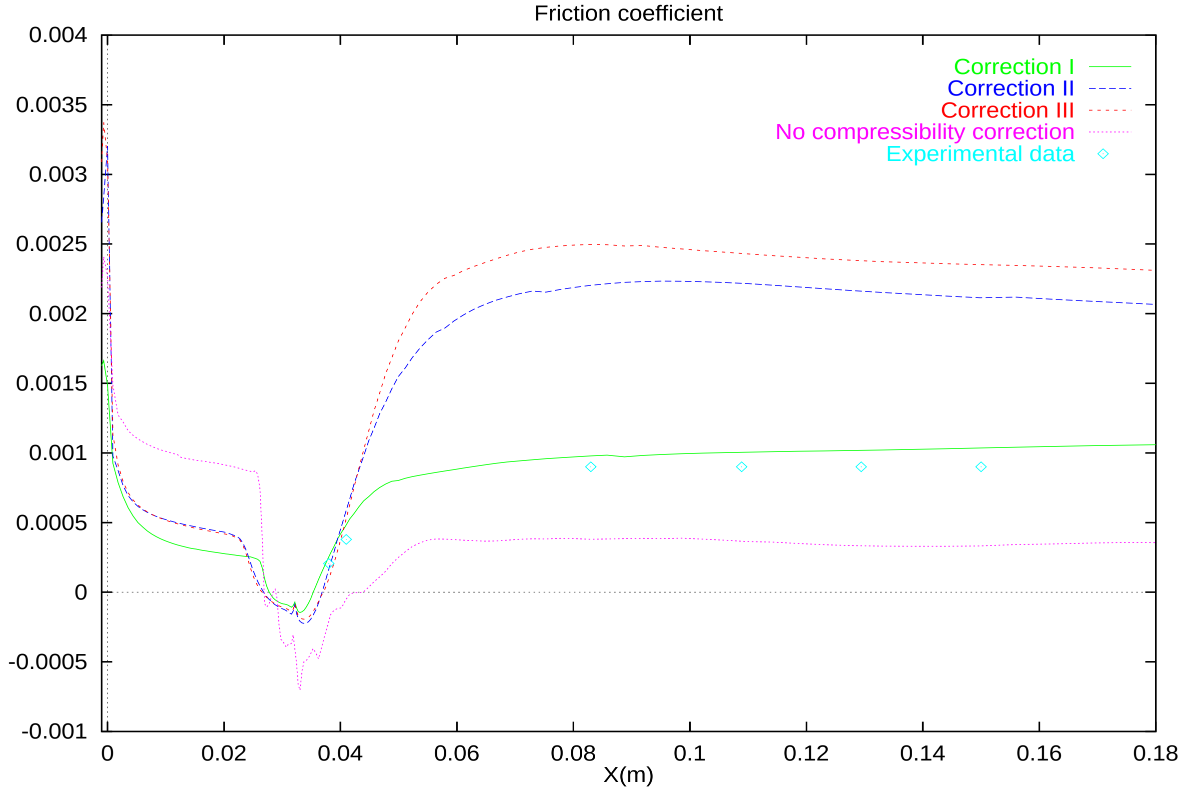
+ linearization permits to recover the boundary integrals,

easy to integrate in commercial software.

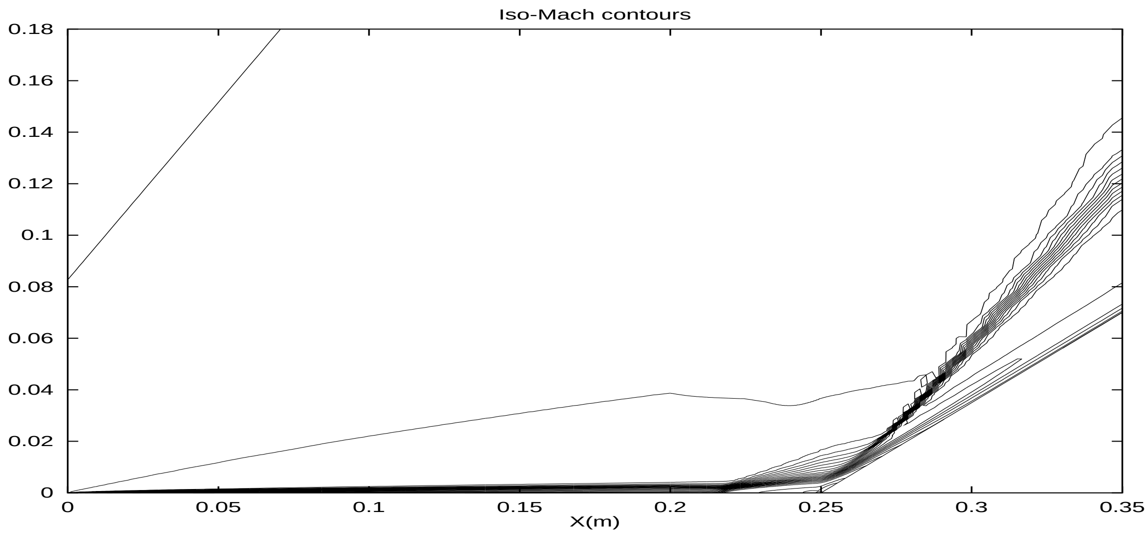
But, incompatible with field turbulence model.



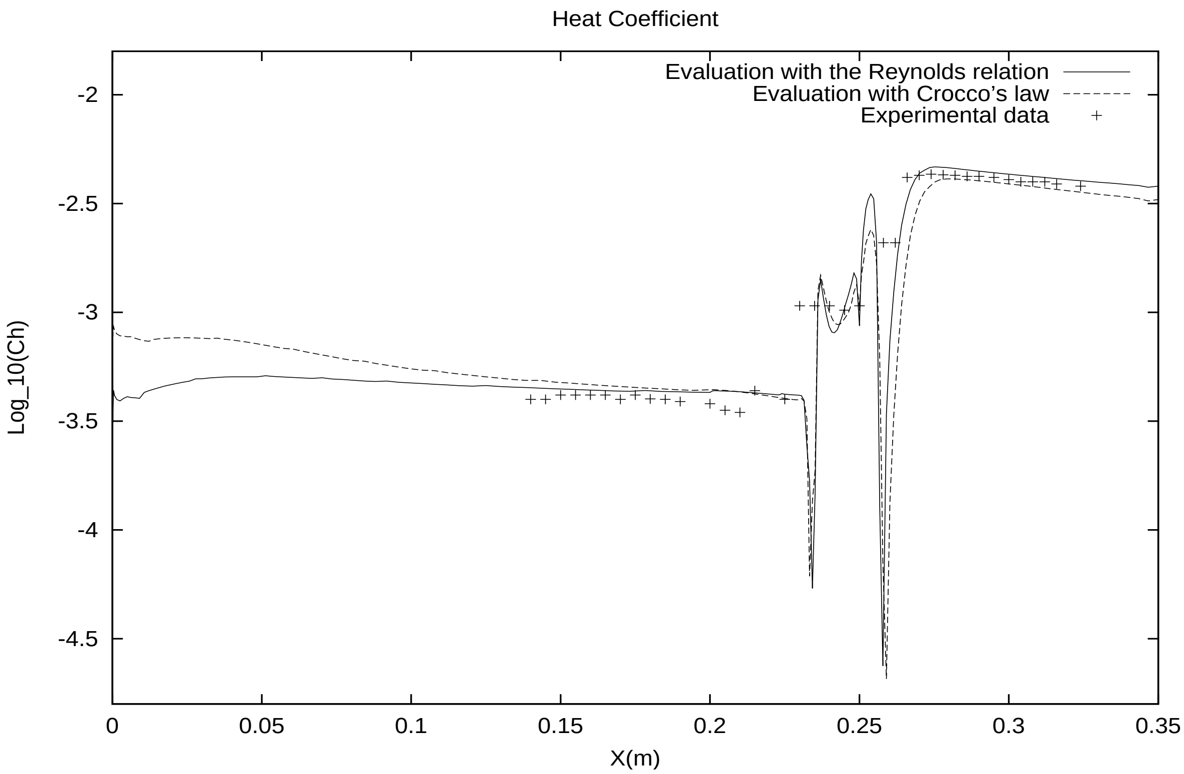
$$M_\infty = 4,\ Re_\infty = 4.5\,.10^7$$

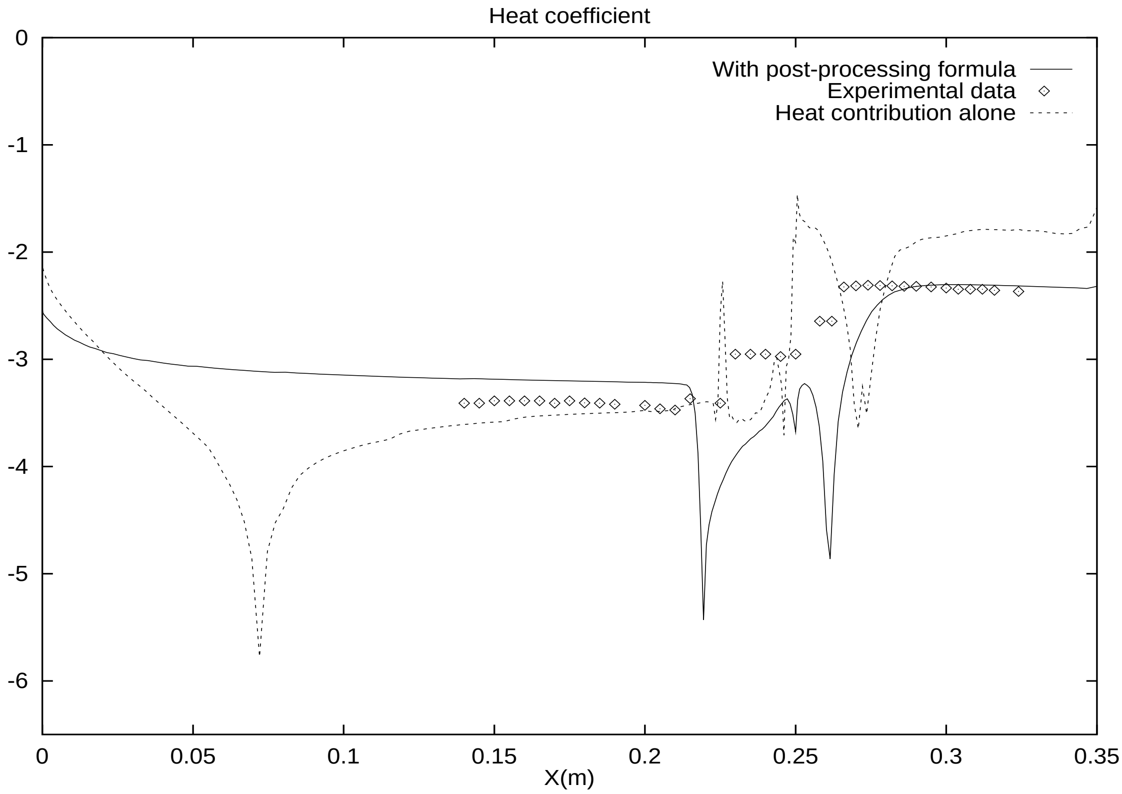


$$C_f$$



$$M_\infty = 5,\ Re_\infty = 4.10^7,\ T_\infty = 83K,$$
$$T_{wall} = 288K$$





Rmq: with wall functions:

$$C_h \sim \frac{\chi \partial_y T|_0}{\rho_\infty u_\infty^3} = \frac{(\chi + \chi_t)\partial_y T|_\delta \quad + \quad u_\delta \quad \rho_w u_\tau^2}{\rho_\infty u_\infty^3}$$

Difficulty with industrial codes.

Wall functions and data assimilation

If full modeling out of reach and if data correlated:

Introduce a priori model for the correction to the wall functions derived for smooth walls:

Relation between the friction along a smooth and rough walls under the same flow conditions

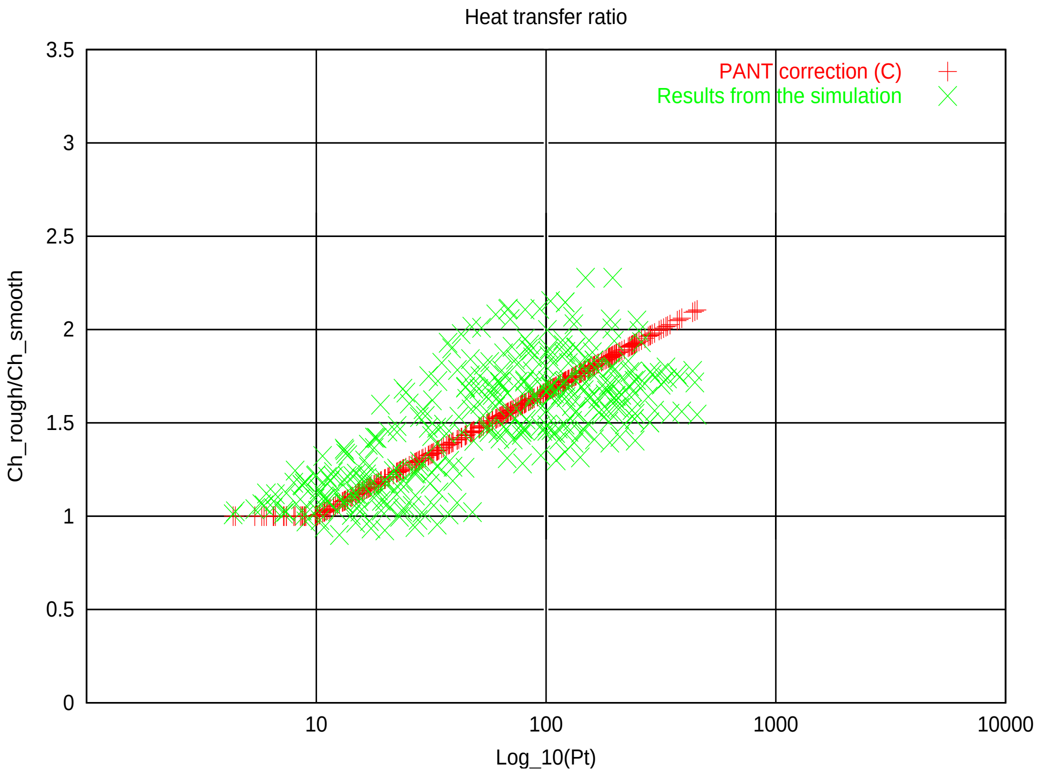
$$P_{C_f} = \frac{C_{f_{rough}}}{C_{f_{smooth}}} = P_2(F, R)$$

F : flow condition variables, R : roughness parameters

P_3 for C_h.

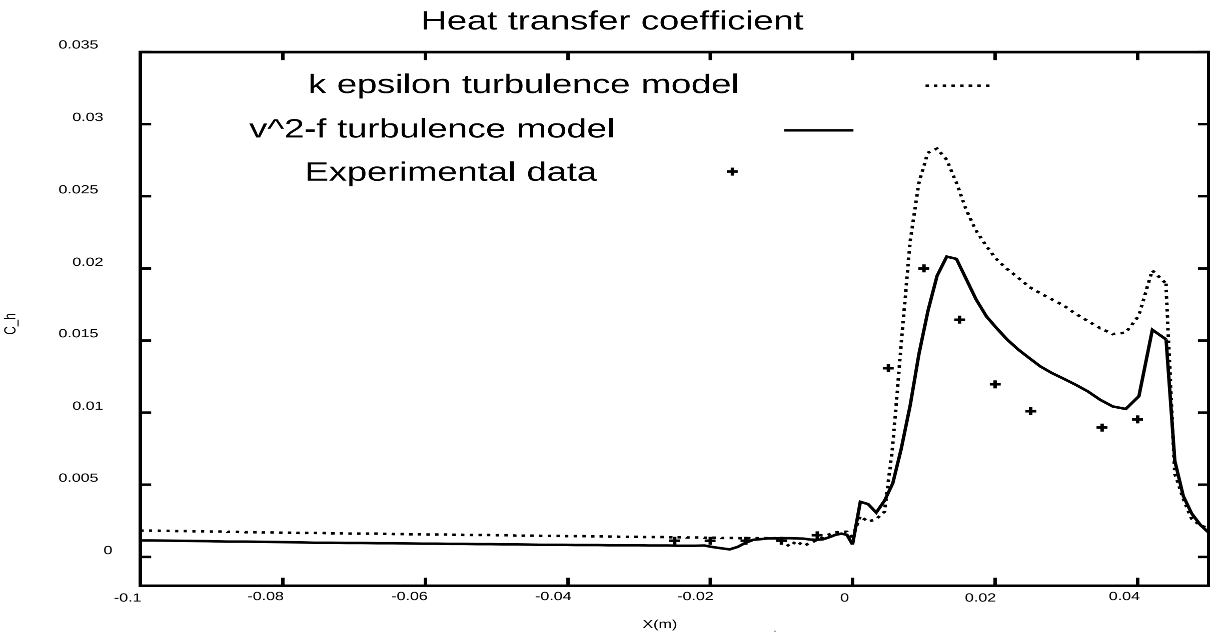
sampling + least square minimization.

These relations are used in the simulation code (not only post processing).

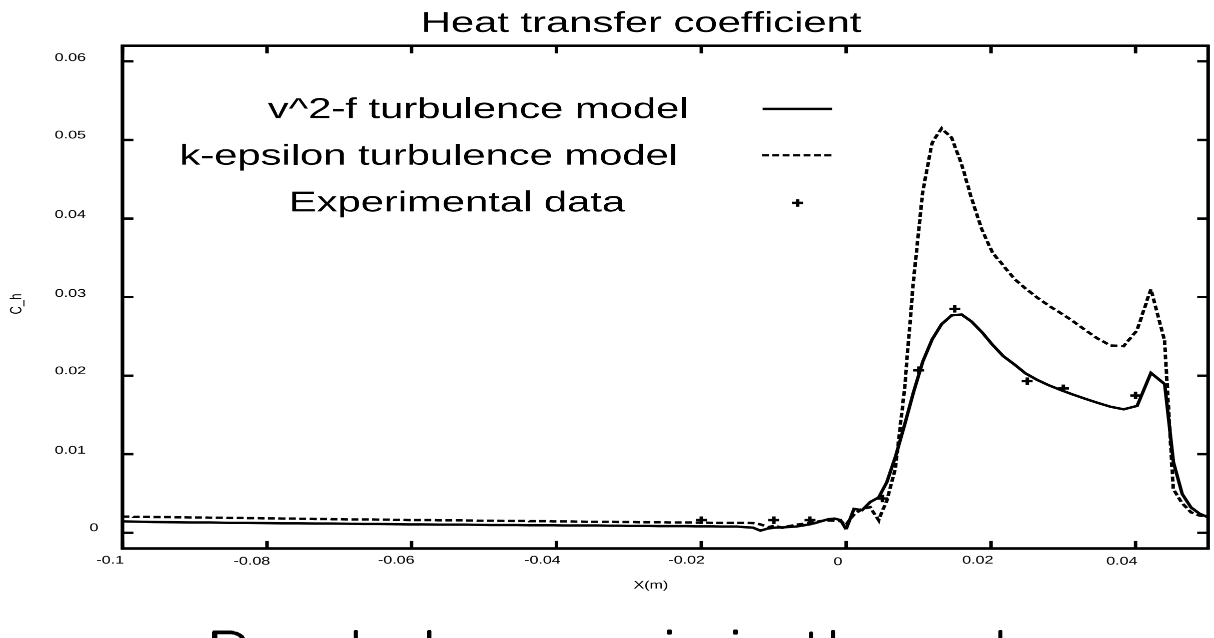


Rmq: Homogenization solving a cell problem: difficulty with complex physics, geometry changes (ablation).

Friction and heat transfert corrections



Smooth



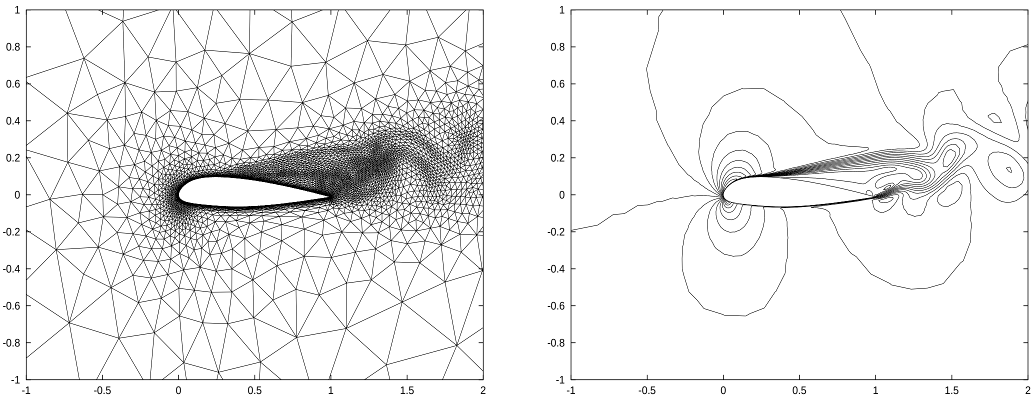
Rough, hypersonic isothermal

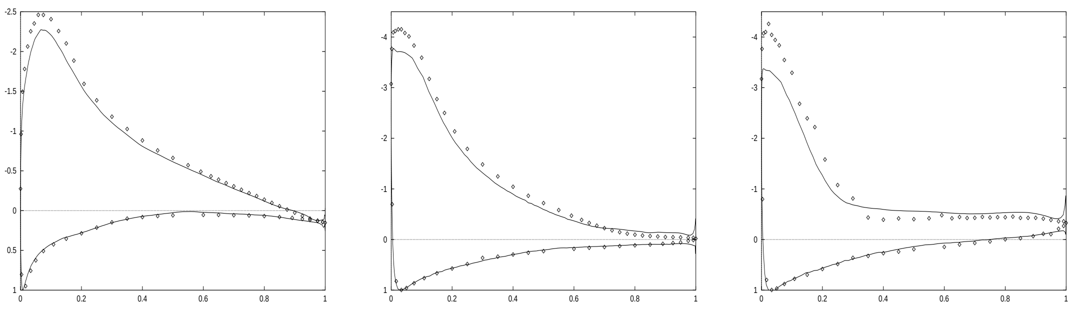
Stall prediction at low speed

accurate time integration

no stall predicted by steady low-Re calculations

wall function suitable for unsteady calculations also





7, 13, 15 degrees

Minimisation problem

Consider $\min_x J(x, q(x), U(q(x)))$,

$x \in \mathcal{O}_{ad} \subset IR^N$: parameterization

$q(x)$: Geometric entities

$U(q(x))$: State variables

Impact of wall functions in simulation and sensitivity evaluation.

Incomplete gradients:

- gradient modeling (as for the state).

- $\tilde{J}'(n, x) \to J'(x), \quad n \to \infty$

Incomplete linesearch.

Injection or transpiration b.c.
Jacques Hadamard (1865-1963)

Moving frame: $u_m.n_m = 0$

Fixed frame: $u_m.n_m = V.n_m$

Suppose $u_m \sim u_f$ then

$$u_f(n_m + n_f - n_f) = V.n_m + |\delta x| o(1)$$

$$\delta x = x_m - x_f$$

Implicit relation on $u_f.n_f$ used in time integration:

$$u_f^{p+1}.n_f = -u_f^p(n_m^{p+1} - n_f) + V^{p+1}.n_m^{p+1}$$

Equivalent transpiration (or injection/suction) b.c.

$\rightarrow$ same analysis on wall function
$u_m.t_m = u_\tau f(y^+) + V.t_m$

$\rightarrow$ will be used in time dependent flow control

Sensitivity analysis

Suppose x a parameter

$$\frac{d}{dx}(u.n) = \frac{\partial u}{\partial x}.n + u\frac{\partial n}{\partial x} \sim u\frac{\partial n}{\partial x}$$

Incomplete sensitivity supposes $\frac{\partial u}{\partial x} << \frac{\partial n}{\partial x}$

u has to be accurate (where needed):

incomplete gradient on a fine mesh rather than 'exact' gradient calculation around a poor state:

multi-level state-sensitivity solution can be a cure:

$$\frac{d}{dx}(u.n) = \mathcal{I}(\frac{\partial u}{\partial x}(\mathcal{P}u_{fine})).n + u.\frac{\partial n}{\partial x}$$

$\mathcal{P}$: fine to coarse restriction

$\mathcal{I}$: coarse to fine projection

Gradient evaluation by incomplete sensitivity

Aim : cheap definition of minimization directions

If $J(x, q(x), U(q)) = f(x, q(x)) \quad g(U(q))$ defined on the shape

then,

$$\frac{dJ}{dx} = \frac{\partial J}{\partial x} + \frac{\partial J}{\partial q}\frac{\partial q}{\partial x} + \frac{\partial J}{\partial U}\frac{\partial U}{\partial x}$$

$$\sim \frac{\partial J}{\partial x} + \frac{\partial J}{\partial q}\frac{\partial q}{\partial x}$$

Changes in the state are negligible compared to those of the geometry for small variations of the domain.

Incomplete Gradient: a simple example

Consider $J = au_x(a)$, (ok: geom times state)

$$E(a,x,u) = u_x - Pe^{-1}\ u_{xx} = 0, \text{ on }]a,1[$$

$$u(a) = 0,\ u(1) = 1$$

$$u(x) \ = \ \frac{\exp(Pe^{-1}\ a) - \exp(Pe^{-1}\ x)}{\exp(Pe^{-1}\ a) - \exp(Pe^{-1})}$$

$$u_x(x) = \frac{-Pe^{-1}\ \exp(Pe^{-1}\ x)}{\exp(Pe^{-1}\ a) - \exp(Pe^{-1})}$$

$$J_a(a) = u_x(a) + au_{xa}(a) = u_x(a)(1 - au_x(a))$$

If $Pe >> 1$, $au_x(a) << 1$, and the state contribution can be neglected.

Analysis holds if $J = f(a)g(u,u_x)$ and also for nonlinear PDEs (Burgers eq.).

IS: Application to channel flows

$$u_{yy} = \frac{p_x}{\nu}, \quad u(-a) = u(a) = 0$$

$$u(a, y) = \frac{p_x}{2\nu}(y^2 - a^2)$$

Flow rate: $J(a, u) = \int_{-a}^{a} u(a, y) dy \quad (= \frac{-2p_x a^3}{3\nu})$

Exact sensitivity:
$\frac{dJ}{da} = \int_{-a}^{a} \partial_a u(a, y) dy \quad (= \frac{-2p_x a^2}{\nu})$

Incomplete sensitivity = 0

Consider: $\tilde{J}(a, u) = \int_{-a}^{a} a u(a, y) dy$

$$\frac{d\tilde{J}}{da} = J + a\frac{dJ}{da} = \frac{-p_x a^3}{\nu}(\frac{2}{3} + 2)$$

Incomplete sensitivity $= \frac{-2p_x a^3}{3\nu}$

Incomplete sensitivity and low-complexity models

From:

$$x \to q(x) \to U(q(x)) \to J(x, q(x), U(q(x)))$$

To:

$$x \to q(x) \to \tilde{U}(q(x))(\frac{U}{\tilde{U}})$$

where $\tilde{U} \sim U$ is obtained using a simple model.

Incomplete sensitivity can be improved by:

$$\frac{dJ}{dx} \sim \frac{\partial J(U)}{\partial x} + \frac{\partial J(U)}{\partial q}\frac{\partial q}{\partial x} + \frac{\partial J(U)}{\partial U}\frac{\partial \tilde{U}}{\partial q}\frac{\partial q}{\partial x}\frac{U}{\tilde{U}}$$

$\tilde{U}$ never used, only $\partial \tilde{U}/\partial q$.

Free choice for low-complexity models. Wall functions are natural $\tilde{U}(y^+)$ candidates.

Example of a low-complexity model

Suppose $U = \log(1 + x)$ and $J = U^2$

$$J' = 2UU' = 2\log(1 + x)/(1 + x)$$

$$\sim 2\log(1 + x)(1 - x + x^2...)$$

Consider $\tilde{U} = x$ valid around $x = 0$

$$J' \sim 2U\tilde{U}' = 2\log(1 + x)$$

On the other hand, after normalizing

$$J' \sim 2U\tilde{U}'(U/\tilde{U})$$

$$= 2\log(1 + x)(\log(1 + x)/x)$$

$$\sim 2\log(1 + x)(1 - x/2 + x^2/3...)$$

which is a better approximation of the gradient.

Wall functions as low-complexity models

In lift and drag sensitivity, we need pressure and friction sensitivity w.r.t shape

$$p_1 n_1 - p_0 n_0 = p_1(n_1 - n_0) + (p_1 - p_0) n_0$$

rmq: I.S. : $\sim p_0(n_1 - n_0)$ $(= 0$ if $n_1 = n_0)$

rmq: we recover $\partial p/\partial n = 0$

Also, for the friction

by definition $u_\tau^2 = \nu \frac{\partial u}{\partial y}|_w = (\nu + \nu_t)\frac{\partial u}{\partial y}|_\delta$ (y normal to the wall)

with I.S. do not use deformation parameterization normal to the wall

use $p_1 = p_0 + \nabla p_0.(x_0 x_1)$ (idem for u but linearizing wall fct.)

Efficiency improvement for a blade

$\eta = \frac{q \triangle p}{\omega T_r}$

constraint on q (inflow rate), ω, Δp (between in and outlet).

But Δp not in the validity domain of IS.

Momentum eq. $\int_\Gamma u(u.n)d\sigma + \int_\Gamma \tau n d\sigma = 0$,

Suppose $n_{i/o} = (\pm 1, 0, 0)$, neglecting viscous terms on in and outlet boundaries and using periodicity:

$$\int_{\Gamma_i} p + \frac{u^2}{2} - \int_{\Gamma_o} p + \frac{u^2}{2} = \int_{\Gamma_w} -p + \nu \frac{\partial u}{\partial n} = C_d$$

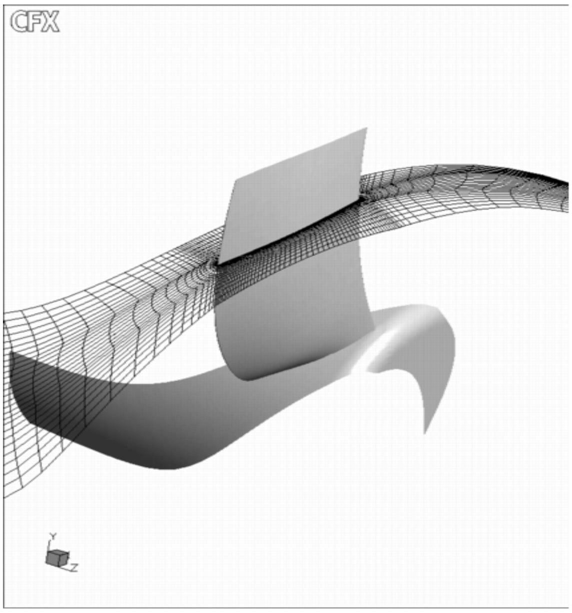
If outlet far enough s.t. $u_o > 0$, from $\nabla . u = 0$ we have:

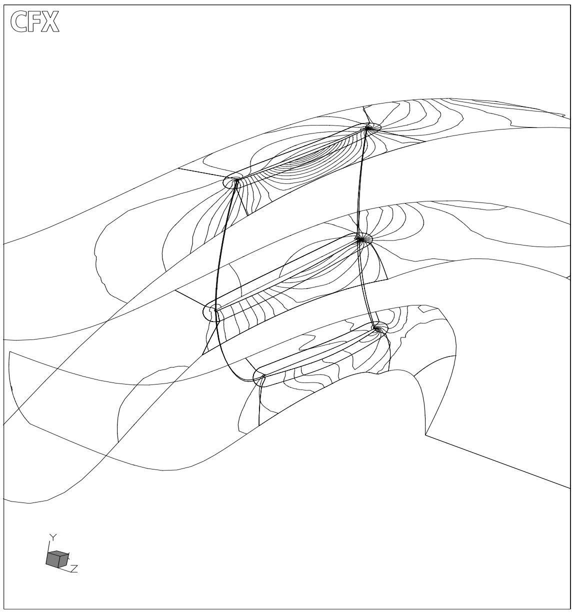
$$\triangle p = C_d$$

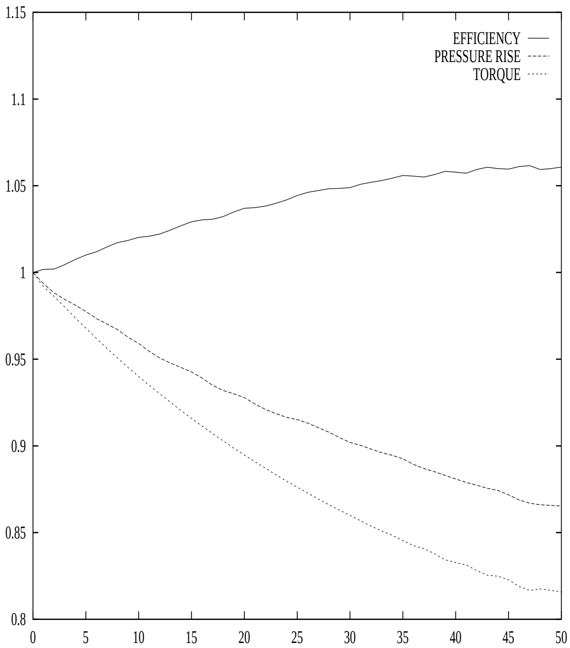
already linearized with wall functions.

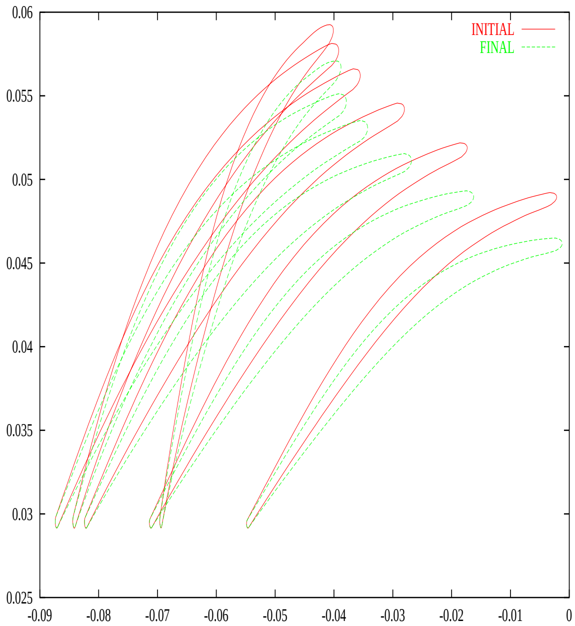
reduce torque @ given drag.

Efficiency improvement for a blade $\eta = \frac{q \triangle p}{\omega T_r}$







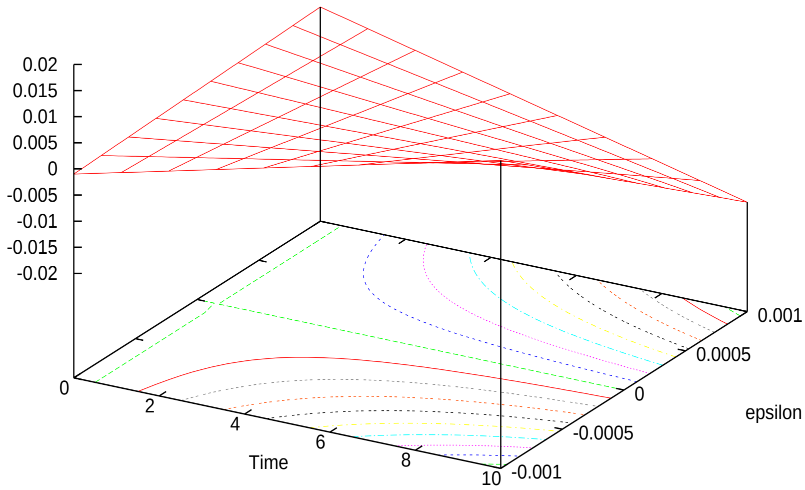


Unsteady configurations

$$J = \epsilon^m \frac{\partial w}{\partial x}(\epsilon), \qquad \begin{cases} \frac{\partial w}{\partial t} - \frac{\partial^2 w}{\partial x^2} = 0 \\ w(\epsilon, t) = w(1, t) = 0 \\ w(x, 0) = sin(\pi \frac{x-\epsilon}{1-\epsilon}) \end{cases}$$

$$w(x,t) = exp(-(\frac{\pi}{1-\epsilon})^2 t) \quad sin(\pi \frac{x-\epsilon}{1-\epsilon})$$

Error between exact and incomplete sensitivity ($|\epsilon| << 1$)



$$\frac{dJ}{d\epsilon}(\epsilon) = \epsilon^{m-1}(m \frac{\partial w}{\partial x}(\epsilon) + \epsilon \frac{\partial^2 w}{\partial x \epsilon}(\epsilon))$$
$$= \frac{\pi \epsilon^{m-1}}{1-\epsilon} exp(-(\frac{\pi}{1-\epsilon})^2 t)(m + \frac{\epsilon}{1-\epsilon}(1 - \frac{2t}{(1-\epsilon)^2}))$$

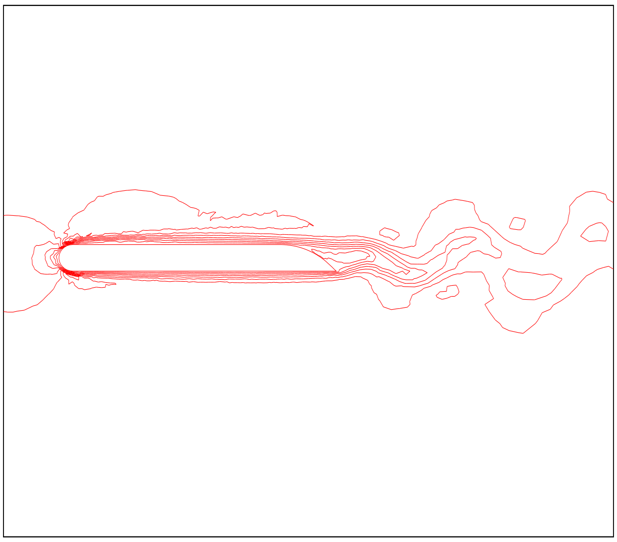
Shape optimization for unsteady flows

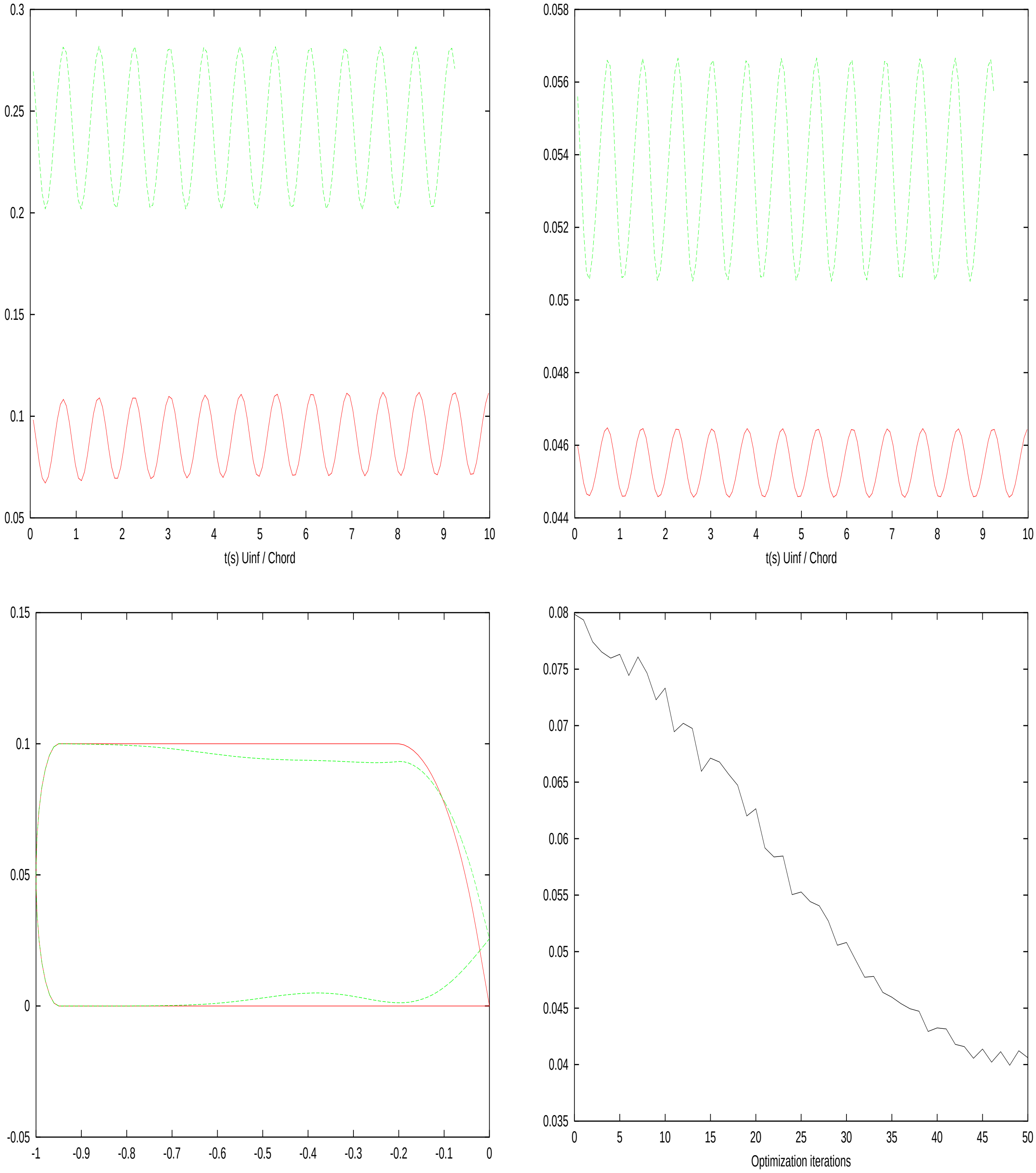
Application to aerodynamic noise reduction reducing lift and drag fluctuations using IS for C_l and C_d:

$$J(x) = \frac{1}{2}\int_0^T (C_l - C_l^{des})^2 + (\partial_t C_l)^2$$

$$\nabla J = \int_0^T (C_l - C_l^{des})\nabla C_l + (\partial_t C_l)\partial_t(\nabla C_l)$$

where ∇C_l is the instantaneous incomplete sensitivity (same for C_d)




t(s) Uinf / Chord
t(s) Uinf / Chord
Optimization iterations

Application to active control using Hadamard b.c.

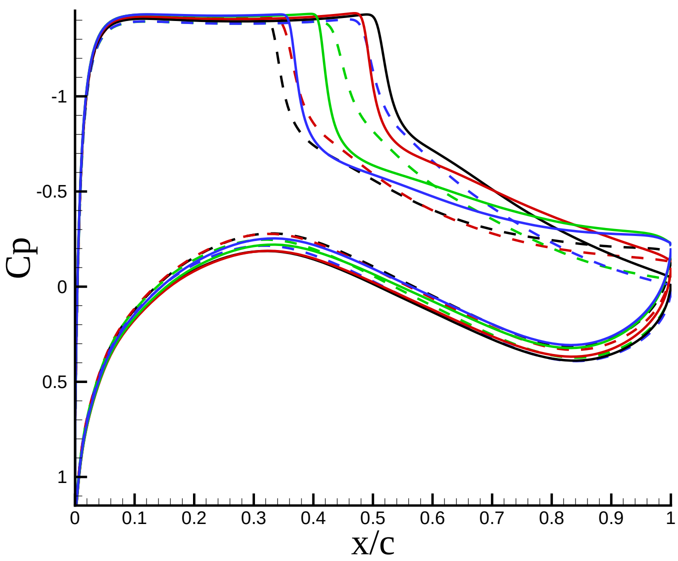
$$u.n(t) = -u(t)\delta n(t) + \delta x(t)/\delta t.n(t)$$

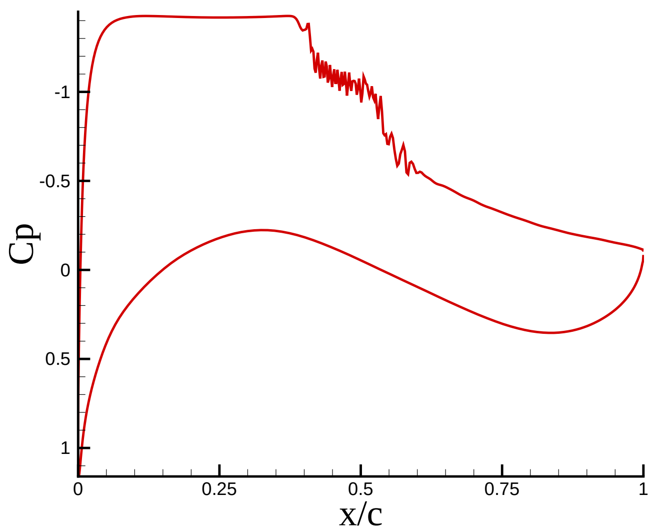
$$\delta x(t) = -\rho \nabla J(t)$$

OAT15A profile:
$Ma = 0.736, Re/m = 4.3610^6, \alpha = 4^o$

Buffeting control: location of actuators, control frequency

Instantaneous pressure coefficients without and after the control is turned on.





Concluding remarks

Global and compatible wall functions if turbulence models valid.

Introduce extra physics through wall functions or update turbulence models (real gas $k - \varepsilon - k_t - \varepsilon_t$).

Impact on meshing and solution (aim explicit time integration).

Low complexity optimization algorithm with incomplete sensitivity updated by wall based models sensitivity.

Current effort: wall functions for external flow calculation with LES using IS and control theory. Difficulty: incompatibility between RANS and LES.