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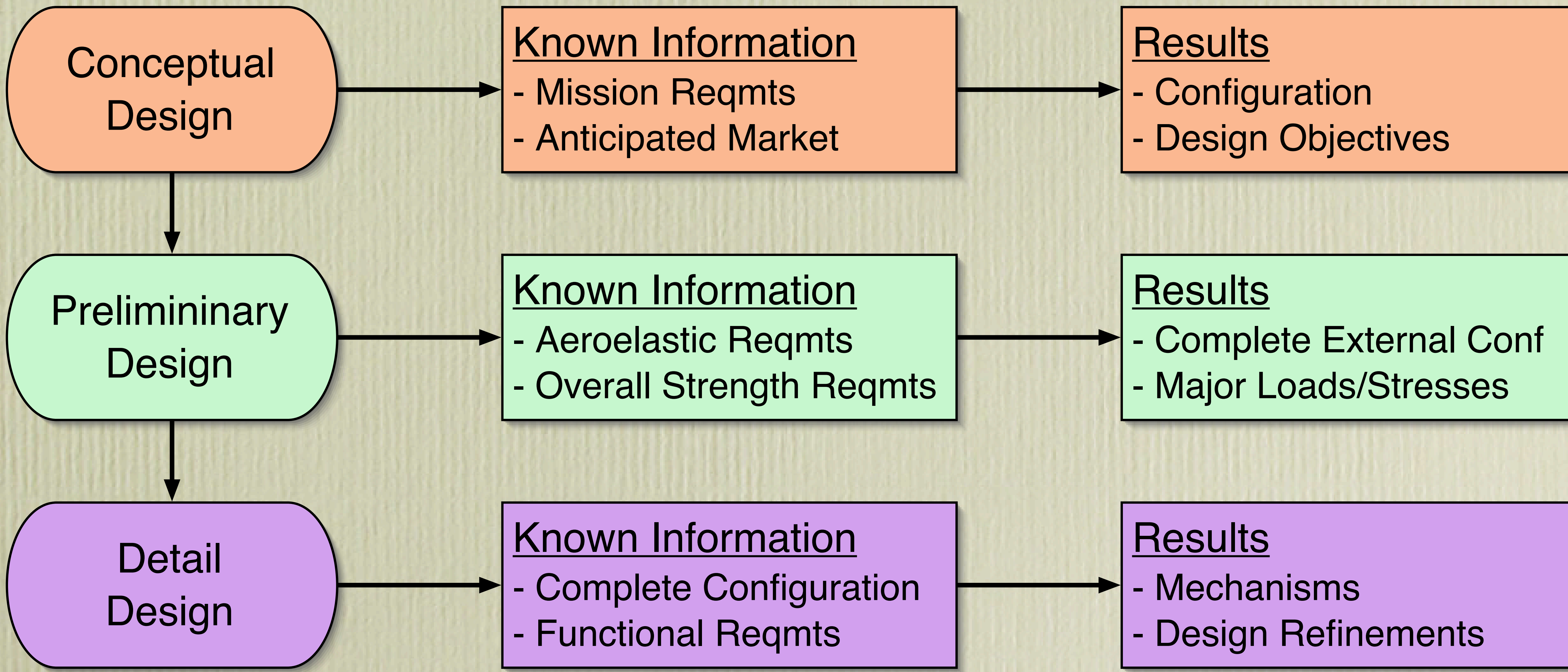
Introduction to Aerodynamic Shape Optimization

1. Aircraft Design Process
2. Aircraft Design Methods
    a. Inverse Surface Methods
    b. Inverse Field Methods
    c. Numerical Optimization Methods

Aircraft Design Process



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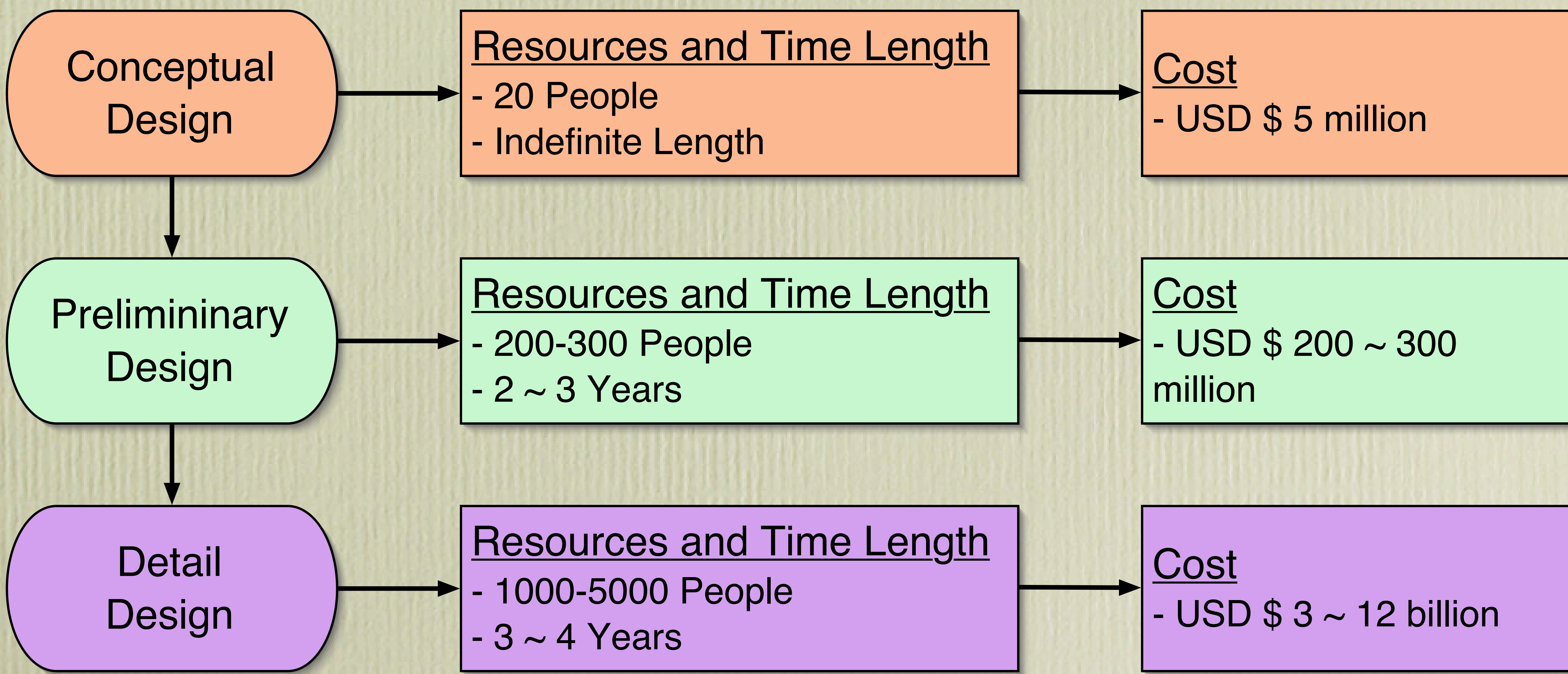
Aircraft Design Process



Medium Size Transport

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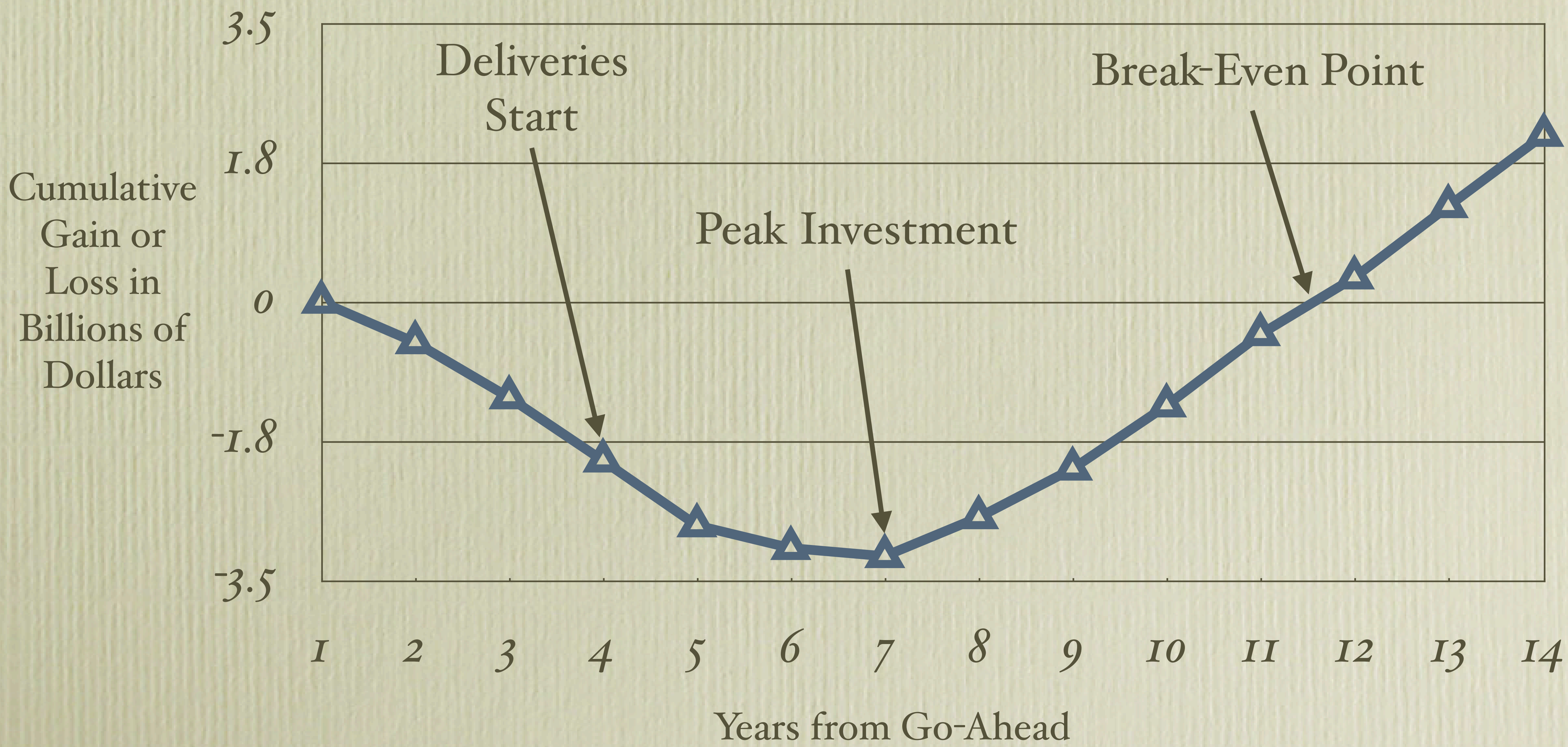
You Bet Your Company

Medium Size Transport Aircraft



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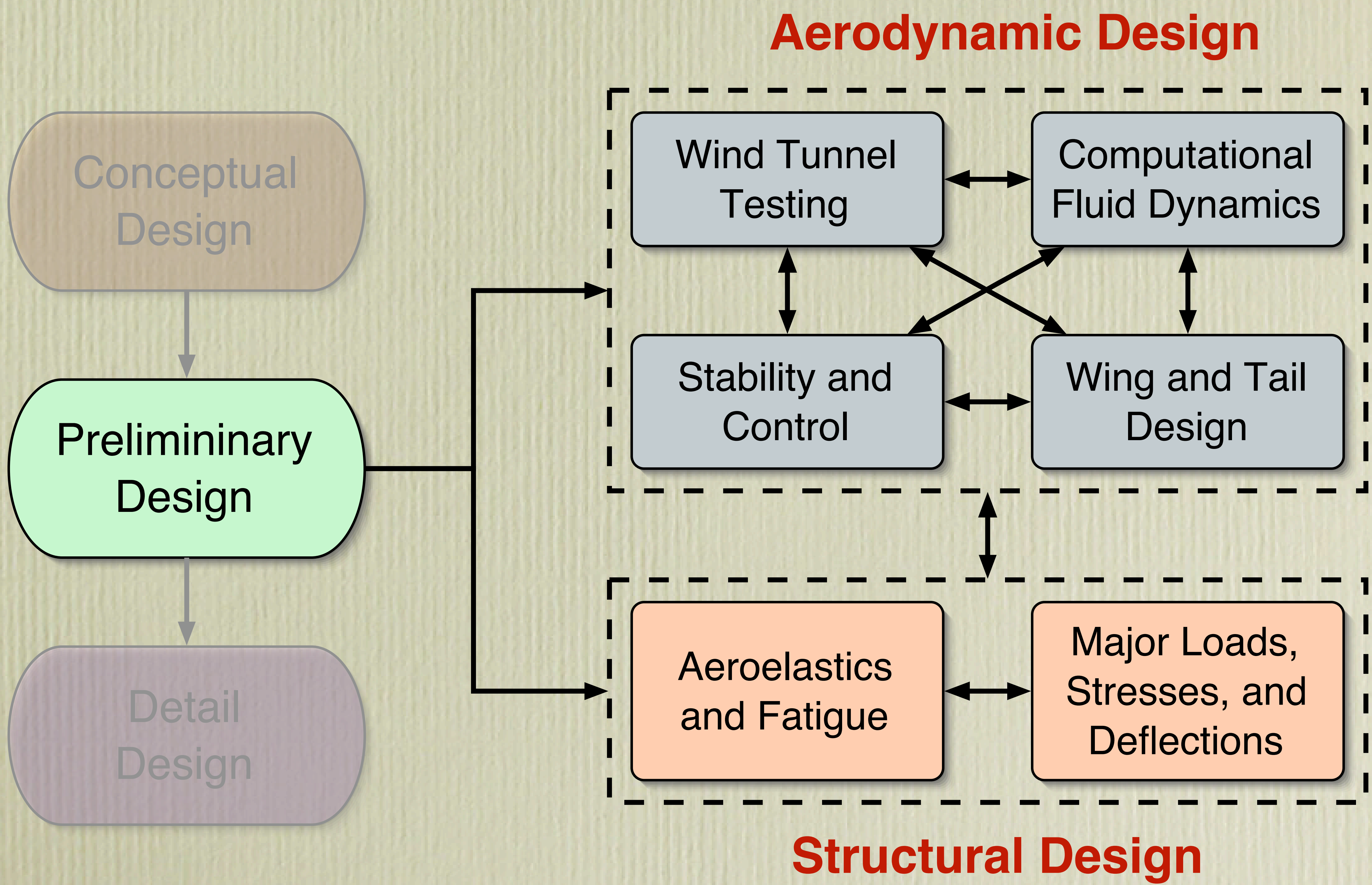
Aircraft Design Process



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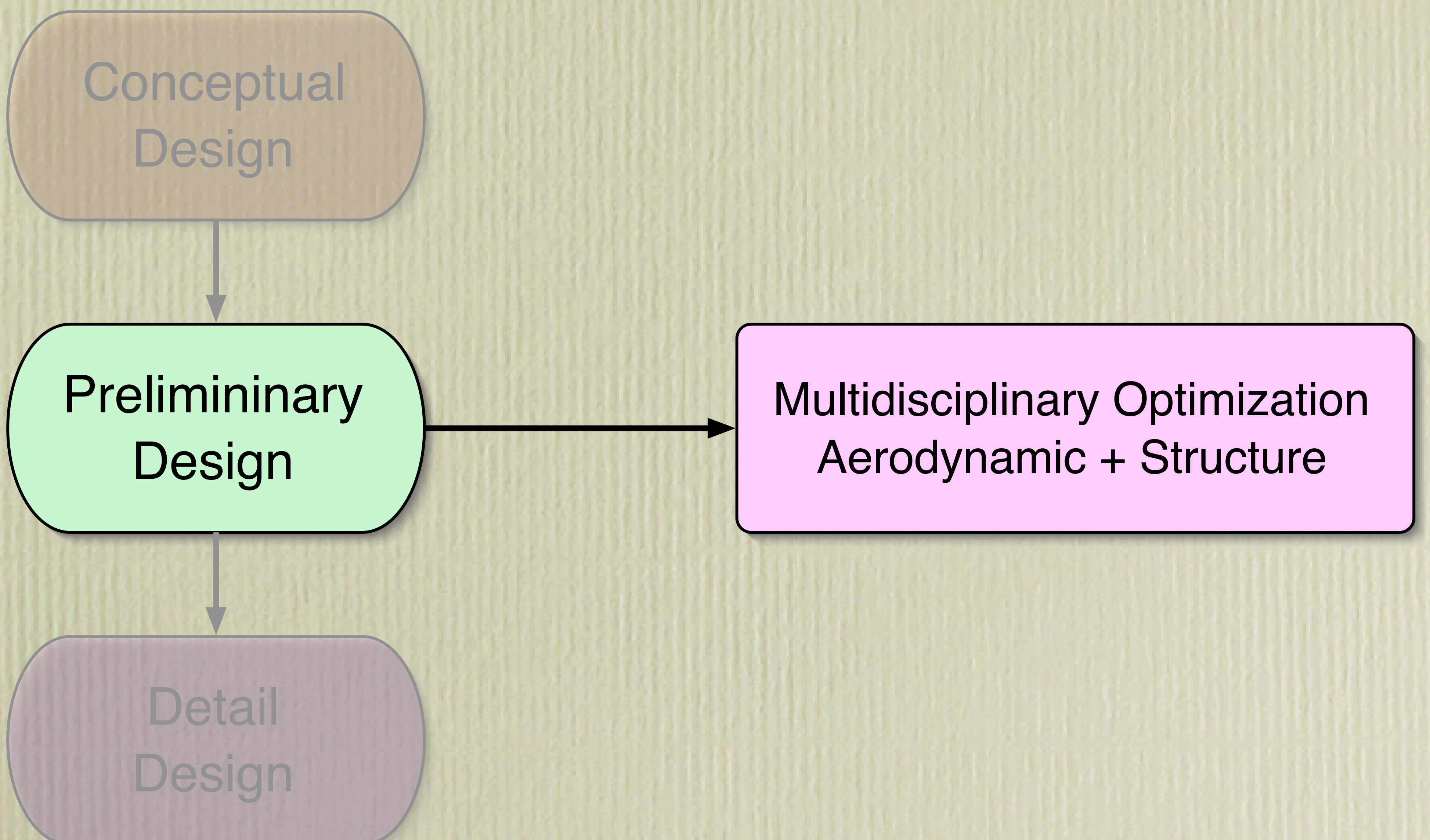
Aircraft Design Process



Introduction to Aerodynamic Shape Optimization

Aerodynamic Design Methods

The **goal** of all aerodynamic design methods, be they experimental, analytical, or computational, is to find a shape which improves an aerodynamic measure of merit while adhering to appropriate constrains

Introduction to Aerodynamic Shape Optimization

Aerodynamic Design Methods

The CFD-based aerodynamic design methods that do exists can be grouped into three basic categories

1. **Inverse Surface Methods**
    a. Incompressible Inviscid Flow
    b. Transonic Potential Flow
    c. Euler and Navier-Stokes
2. **Inverse Field Methods**
3. **Numerical Optimization Methods (Gradient Based)**
    a. Finite Difference
    b. Complex Step
    c. Control Theory Approach

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Inverse Surface Methods

Incompressible Inviscid Flow

- Airfoil or wing shape is computed for a given surface distribution of an aerodynamic quantity.
- *Lighthill* used the method of conformal mapping to solve the two-dimensional inverse pressure problem for the incompressible inviscid flow equations.
  - The airspeed over the profile is given

$$q = \frac{\phi}{h}$$

where ϕ is the velocity potential for flow past a circle and h is the modulus of the conformal mapping function between the circle and the profile.

Since the solution ϕ is also known for incompressible inviscid flow over a circle, then if the analytical mapping is known as well, then the solution over the profile is known.

Inverse Surface Methods

Incompressible Inviscid Flow

- Therefore, if q_d is the desired surface speed, then

$$q_d = \frac{\phi}{h} \qquad h = \frac{\phi}{q_d}$$

-
- The solution to determines the mapping from a circle to the desired shape.
- q is not arbitrary and must satisfy the following constraint:
  - q must attain the freestream value q_∞ in the far field.

$$\int_{-\pi}^{\pi} \log q_o d\theta = 0$$

  -
  - Profile must not produce a gab at the trailing edge.

$$\int_{-\pi}^{\pi} \log q_o \cos\theta d\theta = 0 \qquad \int_{-\pi}^{\pi} \log q_o \sin\theta d\theta = 0$$

Inverse Surface Methods

Transonic Potential Flow

1. *Tranen, T. L.* "A rapid computer aided transonic airfoil design method. AIAA 74-501
    - Tranen replaced the Neumann surface boundary condition $\frac{\partial \phi}{\partial n}$ in an existing CFD potential flow analysis code with a Dirichlet boundary condition ϕ obtained by integrating a desired target velocity distribution.
    - The shape is updated iteratively by the computed normal velocity through the surface.
    - If the target pressure distribution is not realizable the iterations cannot converge.

Inverse Surface Methods

Transonic Potential Flow

2. *McFadden* and *Garabedian* extended *Lighthill's* method.

- The flow equation is first solved for a given mapping h_o.
- Then an updated mapping is determined by setting $q = q_d$.
- The flow equation is then solved for this new mapping, h_1 , and the process is repeated.
- It does not require a modification to a Dirichlet boundary condition at the surface. Therefore it retains a valid solution during the entire design process.

Introduction to Aerodynamic Shape Optimization
Inverse Surface Methods

Euler and Navier-Stokes

- *Campbell*. An approach to constrained aerodynamic design with application to airfoils. NASA Technical Paper 3260, Langley Research Center, November 1992.
  - The difference between the target and actual pressures is translated into surface changes through the use of the relationship between:
    - surface curvature and pressure for subsonic flow,
    - and, surface slope and pressure for supersonic flow.
  - The method has been very successful for 2D Euler and Navier-Stokes equations. The method suffers from 3D flows with cross-flow character since the surface curvatures and slopes are calculated plane by plane

$$\Delta n + \beta_1 \frac{\partial}{\partial x}\Delta n + \beta_2 \frac{\partial}{\partial y}\Delta n + \beta_3 \frac{\partial^2}{\partial x^2}\Delta n + \beta_4 \frac{\partial^2}{\partial y^2}\Delta n = \beta_5 \Delta c_p$$

$\Delta n =$ local normal surface displacement

$\beta_i =$ user specified quantities

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Inverse Field Methods

- Designs are based upon objectives or constraints imposed on the configuration surface but everywhere in the flow field.
  - *Garabedian* and *Korn*.
    - Hodograph transformation.
    - Method can guarantee that no shock will occur in the flow field.
    - Has been successful in the development of airfoils displaying shock-free transonic flows.
    - Its major disadvantage is that hodograph transformations are not applicable to three dimensions.
  - Sobieczky. Fictitious gas method.

Disadvantage of Inverse Surface and Field Methods is that the objective of a target pressure distribution is built directly into the design process and thus cannot be changed to any arbitrary objective function.

Numerical Optimization Methods

Gradient Based

- Concept of a gradient.
  - Define the geometry by weights, α_i and shape functions, b_i so that the shape can be represented by

$$f(x) = \sum_{i=1}^{n} a_i b_i(x)$$

  - Then a cost function I is selected. I is regarded as a function of the parameters α_i
  - Then,

$$\delta I = \sum_{i=1}^{n} \frac{\partial I}{\partial \alpha_i} \delta \alpha_i \quad \text{where, } \frac{\partial I}{\partial \alpha_i} \text{ is a gradient}$$

The gradient vector $\frac{\partial I}{\partial \alpha_i}$ may now be used to determine a direction of improvement.