Symmetric random walks in random environments

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In this talk I will survey recent progress on the *random conductance model.*

Let $(\mathbf{Z}^d, \mathbf{E}_d)$ be the Euclidean lattice, and let μ_e, $e \in \mathbf{E}_d$ be i.i.d.r.v on $[0,\infty)$. Let $Y = (Y_t, t \in [0,\infty), P^x_\omega, x \in \mathbf{Z^d})$ be the continuous time Markov chain with jump probabilities from x to y proportional to μ_{xy}. Thus Y has generator

$$Lf(x) = \gamma_x^{-1} \sum_{y \sim x} \mu_{xy}(f(y) - f(x)),$$

where γ is a 'speed' measure. Two natural choices for γ are $\gamma_x = 1$ for all x, and $\gamma_x = \mu_x = \sum_{y\sim x} \mu_{xy}$. In the first case the mean waiting time at x before a jump is $\mu_x = \sum_y \mu_{xy}$, and in the second it is 1. We can call these respectively the 'variable speed' and 'fixed speed' continuous time random walks – VSRW and CSRW for short. A special case of the above is when $\mu_e \in \{0,1\}$, and so Y is a random walk on a (supercritical) percolation cluster. In this talk I will discuss recent work, by myself and others, on invariance principles for Y, and Gaussian bounds for the transition densities

$$q^\omega_t(x,y) = \gamma(y)^{-1} P^x_\omega(Y_t = y).$$

I will also discuss the connection with 'trap' models and 'ageing' phenomena.