

Animal Behavior & Geometry

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This is joint work with:



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Our story begins with...



He also wrote about stereotypical alarm behavior of fresh water fish that live in shoals in the paper "The Psychology of Fish Swarms" in 1938.

Karl von Frisch

von Frisch received the Nobel Prize in Psychology or Medicine in 1973 for the work described in his book "The Dancing Bees" from 1927.



A Brief Overview...

- von Frisch observed that injured minnows release a substance he called "Schreckstoff" which induces panic among nearby fish.
- Biologists believe this developed as an anti-predator mechanism to alert other members of a shoal to danger.
- Schreckstoff can be extracted from the skin and used for experiments, though its active ingredients are not well understood.
- From the 1970's it was believed that Shreckstoff was only produced by members of the Ostariophysi superorder of freshwater fish.

Recent Development

Mathuru discovered the same alarm behavior in medaka fish, which belong to the Beloniformes order of fish.



Ajay S. Mathuru. Conspecific injury raises an alarm in medaka. Scientific Reports, 2016; 6: 36615.

Long-term Goal

Understand the evolution of alarm response and its relationship to other innate behaviors.

Idea

Try to understand the geometry of the motion of these fish.





Alarm response in Zebrafish (indicator shows alarm substance delivery)



Alarm response in Medaka (indicator shows alarm substance delivery)

Why Schreckstoff and why fish?

- Innate responses to danger are ubiquitous in vertebrates.
- These fish can be bred so that their neural activity can be monitored without invasive procedures that might alter their natural behavior.
- Neural activity can be visualized using a genetically engineered protein that converts voltage change to fluorescence change.



Akerboom J, et. al. J. Biol. Chem. 284 (10): 6455-64.



Transgenic fish expressing GCaMP in the central nervous system.

The Experiment

- A fish is placed by itself in a small tank and left to adjust to the new environment.
- After 2 minutes an alarm (or control) substance is carefully released into the tank.
- The movement of the fish is tracked for a duration of 10 minutes (total) using the software *idTracker*.

http://www.idtracker.es

 A fish exposed to Schreckstoff will typically exhibit a short period of *darting* (increased velocity and rapid changes in direction) followed by a moderate period of *stillness* and gradual movement toward the bottom of the tank.

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The Experiment



Data from six Schreckstoff (red) and six control (blue) fish and their respective averages (bold).

Quantifying Behavior

- We want to be able to do more than simply say whether or not a fish was exposed to Schreckstoff.
- We want a metric that measures the similarity/ dissimilarity in behavior between two trajectories.
- We want to establish correspondences between trajectories that align key behavioral features.
- We want to identify meaningful correspondences between observed behaviors and neural activities.

Quantifying Behavior



Quantifying Behavior

• The trajectory of each fish is a smooth parametric curve ${\cal C}$ in \mathbb{R}^3 given by a function

$$\mathbf{r}(t) = (x(t), y(t), z(t))$$

over a time interval $t \in [t_{\alpha}, t_{\omega}]$.

• Between two curves, C_1 and C_2 , we want to find a correspondence that matches points with similar behaviors.

Curve Alignment



Source: Derivative Dynamic Time Warping

Curve Alignment

• This is a large research area with lots of active work.

Rigid Motions

- Extrinsic Geometry
- Signature Verification

Deformations

- Intrinsic Geometry
- Character Recognition

Richard Kunbell

Lichard Kurball



Some Considerations

- 1. We have no control over the position/orientation of the fish at the start of each experiment.
- 2. Depending on the initial position/orientation, the amount of time it takes for the Schreckstoff to reach the fish will vary.
- 3. We are more interested in how the fish *behaves* at a particular location (or moment in time) than where it is located.

We are primarily interested in intrinsic geometry.

Differential Geometry

Consider the motion of a fish along the parametric curve $\mathbf{r}: [t_{\alpha}, t_{\omega}] \to \mathbb{R}^3$.

- If $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ are parallel, then the fish is traveling in a straight line not much interesting is happening.
- If $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ are linearly independent, then
 - the unit tangent vector $T(t) = \frac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||}$
 - the unit normal vector $N(t) = \frac{T'(t)}{||T'(t)||}$ are orthogonal, and together with



• the unit binormal vector $B(t) = T(t) \times N(t)$

form an orthonormal basis for \mathbb{R}^3 called the *Frenet-Serret frame*.

We can understand the local behavior of the fish by analyzing how this frame changes with respect to time.

Differential Geometry

Theorem: (Frenet-Serret)

$$\frac{d}{dt} \begin{bmatrix} T\\N\\B \end{bmatrix} = ||\mathbf{r}'(t)|| \begin{bmatrix} 0 & \kappa & 0\\ -\kappa & 0 & \tau\\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T\\N\\B \end{bmatrix}$$

• $\kappa = \frac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$ is the *curvature* of the **r** at time *t*. $\det(\mathbf{r}'(t) \mathbf{r}''(t) \mathbf{r}'''(t))$

• $\tau = \frac{\det(\mathbf{r}'(t), \mathbf{r}''(t), \mathbf{r}''(t))}{||\mathbf{r}'(t) \times \mathbf{r}''(t)||^2}$ is the *torsion* of **r** at time *t*.

The motion of the fish is completely described by $||\mathbf{r}'(t)||$, κ , and τ .

• Given two parametric curves, C_1 and C_2 , we want to match points of C_1 with the points of C_2 that have similar real-time velocities, curvatures, and torsions, and vice versa.



Source: Derivative Dynamic Time Warping

- Given two parametric curves, C_1 and C_2 , we want to match points of C_1 with the points of C_2 that have similar real-time velocities, curvatures, and torsions, and vice versa.
- An *alignment* between \mathcal{C}_1 and \mathcal{C}_2 is a function of the form

$$\varphi: [u_{\alpha}, u_{\omega}] \to [t_{\alpha}, t_{\omega}]^2 \qquad \varphi(u) = (\varphi_1(u), \varphi_2(u))$$

where $\varphi_i(u_\alpha) = t_\alpha$, $\varphi_i(u_\omega) = t_\omega$, and φ_i is monotone increasing and differentiable for each $i \in \{1, 2\}$.

• An alignment matches every point on C_1 with at least one point on C_2 and vice versa in a way that preserves chronology, but allows for some stretching and compressing (even pausing) of time.

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Source: Derivative Dynamic Time Warping

We want to find an alignment that does the best job of matching points with similar real-time velocities, curvature, and torsion.

The *behavioral distortion energy* of φ is given by

$$E_{bd}(\varphi) = \sqrt{\int_{u_{\alpha}}^{u_{\omega}} \left(\left| \left| \mathbf{r}_{1}'(\varphi_{1}(u)) \right| \right| - \left| \left| \mathbf{r}_{2}'(\varphi_{2}(u)) \right| \right| \right)^{2} du}$$

$$+\sqrt{\int_{u_{\alpha}}^{u_{\omega}} \left(\kappa_1(\varphi_1(u)) - \kappa_2(\varphi_2(u))\right)^2 du}$$

$$+\sqrt{\int_{u_{\alpha}}^{u_{\omega}} \left(\tau_1(\varphi_1(u)) - \tau_2(\varphi_2(u))\right)^2 du}$$

- Finding an optimal alignment φ^* corresponds to solving the minimization problem

$$\varphi^* = \underset{\varphi}{\operatorname{argmin}} E_{bd}(\varphi)$$

• If an optimal alignment exists, we can define a distance between C_1 and C_2 by

$$d(\mathcal{C}_1, \mathcal{C}_2) = E_{bd}(\varphi *)$$

 It's not clear that there is always an optimal alignment, but there is a family of fast heuristics for approximating them that use dynamic programming.

Dynamic Time Warping

Show that the distance function has *optimal substructure*:

For every $u_1 < u_2 < u_3$,

 $d(\mathcal{C}_1|_{[\varphi_1^*(u_1),\varphi_1^*(u_3)]}, \mathcal{C}_2|_{[\varphi_2^*(u_1),\varphi_2^*(u_3)]}) = d(\mathcal{C}_1|_{[\varphi_1^*(u_1),\varphi_1^*(u_2)]}, \mathcal{C}_2|_{[\varphi_2^*(u_1),\varphi_2^*(u_2)]})$

+
$$d(\mathcal{C}_1|_{[\varphi_1^*(u_2),\varphi_1^*(u_3)]}, \mathcal{C}_2|_{[\varphi_2^*(u_2),\varphi_2^*(u_3)]})$$



Source: Frénay et al. Curve Alignment: Theory and Applications. UCL/ICTEAM - MLG, 2012.

Heat Maps

There are occasions when neuroscientists are interested in the general location of the fish.



Surface Alignment



100 500 80 -400 - 300 - 300 -Juedneuc 60 40 · 20 -- 100 0 -0 25 50 75 100 125 150 175 200

Heatmap during freezing

Heatmap after adding SS and before freezing







Thank you!

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