Signal Sampling and Reconstruction on Spatially Distributed Networks

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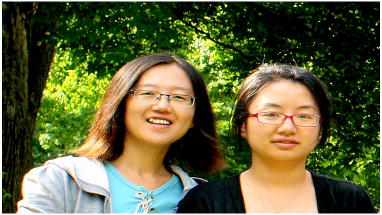
- Thank the organizer for the invitation to attend this workshop and give a talk on mathematical framework for spatially distributed networks.
- Thank professor Waishing Tang for his invitation to visit Department of Mathematics, National University of Singapore.
- This talk is based on a joint work with Yingchun Jiang and Cheng Cheng; more



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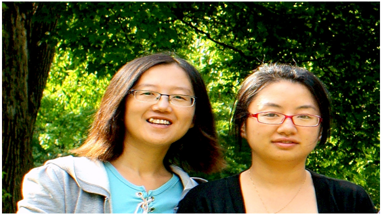

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Outline

- Spatially distributed networks and their topologies
- Graph description for signal sampling and reconstruction systems
- Sensing matrices and local stability criterion
- Distributed implementation for signal reconstruction algorithms

Part I: Spatially distributed networks (SDN)

Spatially distributed network (SDN) contains large amount of small devices with sensing, data processing, and telecommunications capabilities feasible.

- sensing capability
- limited telecommunication range
- certain data processing ability (computing power)

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Part I: SDNs and our study

- Our model of SDNs: wireless sensor networks with remote sensors deployed over a region and communication available only within a spatial range; smart power grids with sparse interconnection topologies, multi-agent systems with nearest-neighbor coupling structures, and perhaps image denoising by patch-based local models.
- Our study: mathematical framework on SDN adaptive to signal sampling and reconstruction.

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- Our study: mathematical framework on SDN adaptive to signal sampling and reconstruction.

Part I: Advantage of SDS

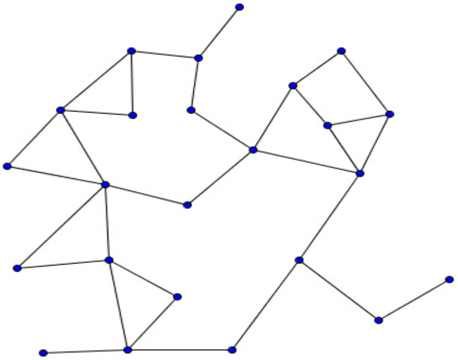
An SDN could give unprecedented capabilities for signal sampling and reconstruction, especially when

- creating a data exchange network requires significant efforts (due to physical barrier such as interference in relatively inexpensive infrared lasers)
- establishing a centralized processor presents the daunting challenge of processing all the information (such as big-data problem, reliable (wired) communication unavailable).

Part I: Graph description of SDNs

We describe the topology of an SDN by a graph $\mathcal{G} = (G, S)$,

- a vertex represents a sensing device
- an edge between two vertices means that direct communication link to exchange messages exists between those two devices.



Part I: What we want to do for SDNs?

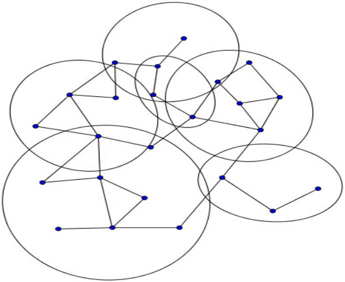
We study SDNs for signal sampling and reconstruction. What we need and what we can do?

- Stability (robust against sampling error)
- Local and real-time reconstruction (signals reconstructed from neighboring sampling data approximately, signals on a manifold)
- Limited computation and communication for sensing devices (Depending only on neighbors, not the size of the system)
- Feasibility (supplement, replacement, and impairment of sensing devices)

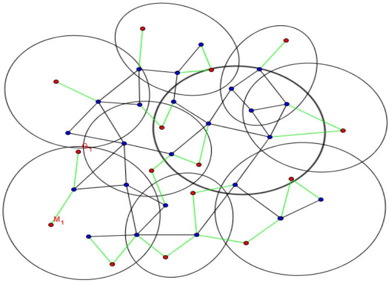
Our strategy

How to solve for spatially distributed sampling/reconstruction systems:

- divide an SDN into overlapped subsystems with limited size, and use the uniform stability of spatially distributed subnetworks to study robustness of the SDN



- Reconstruct signal locally via subnetworks and then stitch the patch solutions of subsystems to form an approximation of the true signal.



Signal sampling and reconstruction on an SDN?

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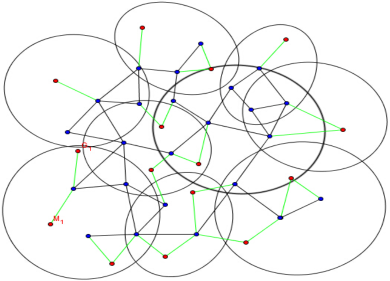


Signal sampling and reconstruction on an SDN?

Part II: Graph structure of sampling system

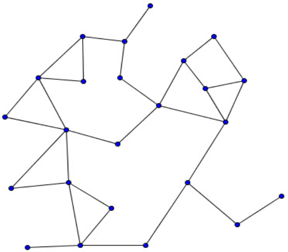
Question: How to describe a spatially distributed sampling system?
Answer: connected simple graph $\mathcal{G} := (G, S)$

- Vertex: spatial location of a device with sensing, computing and telecommunications capabilities.
- Edge: direct communication exists between devices at two vertices(spatial locations).
- connected: communication across the entire network
- simple (undirected, unweighted, no graph loops or multiple edges).

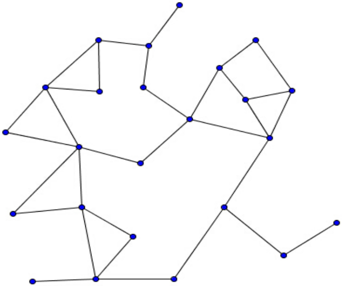
Part II: Graph structure of sampling system

simple graph: undirected, unweighted, no graph loops or multiple edges.

- undirected: bi-directional direct communication links
- unweighted: evices with almost same communication specification
- no graph loops: No communication within each device
- no multiple edges: no multiple direct communication channels between devices exist



Part II: Graph structure of sampling system



- Geodesic distance: $\rho_{\mathcal{G}}(\lambda, \lambda')$ is the number of edges in a shortest path connecting two distinct vertices.
- Counting measure $\mu_{\mathcal{G}}$: $\mu_{\mathcal{G}}(F) := \sharp(F)$ for $F \subset G$.
- Measure of a ball $\mu_{\mathcal{G}}(B_{\mathcal{G}}(\lambda, r))$?

Part II: Dimension and density of sampling system

- Polynomial growth: if there exist positive constants $D_1(\mathcal{G})$ and $d(\mathcal{G})$ such that

$$\mu_{\mathcal{G}}(B_{\mathcal{G}}(\lambda, r)) \le D_1(\mathcal{G})(1+r)^{d(\mathcal{G})} \ \text{ for all } \lambda \in G \text{ and } r \ge 0.$$

- Beurling dimension:

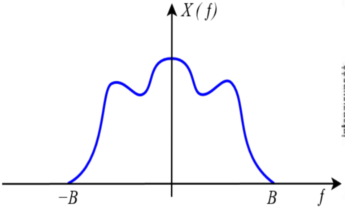
$$d(\mathcal{G}) = \limsup_{r\to\infty} \sup_{\lambda\in G} \frac{\ln \mu_{\mathcal{G}}(B_{\mathcal{G}}(\lambda, r))}{\ln(1+r)}$$

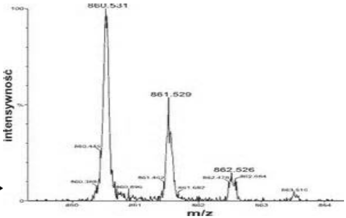
- Maximal sampling density:

$$D_1(\mathcal{G}) = \sup_{r\ge 0} \sup_{\lambda\in G} \frac{\mu_{\mathcal{G}}(B_{\mathcal{G}}(\lambda, r))}{(1+r)^{d(\mathcal{G})}}.$$

Part III: What kind of signals we want to study?

- spatial signals $f = \sum_{i \in V} c_i \varphi_i$, where φ_i is the generating signal at innovative position $i \in V$, and amplitudes $c_i, i \in V$, are bounded.[1]
- bandlimited signals, spline signals, global positioning system, ultra wide-band communication, mass spectrometry.





[1] M. Vetterli, P. Marziliano, and T. Blu, *IEEE Trans. Signal Proc.*, 2002. and Q. Sun, *Adv. Comput. Math.*, 2008.

Part III: Signals and SDNs

Question: How to connect signal to sampling graph?

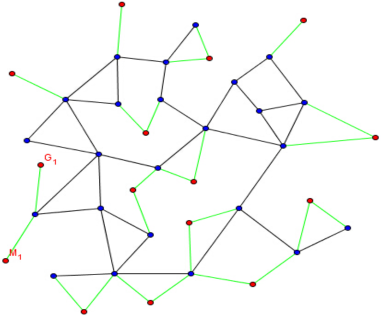
- For every innovative position $i \in V$, we associate it with locations $\lambda \in G$ of principal sensing devices.
- Let T contain all pairs of innovative positions and locations of their principal sensing devices, and let $T^* = \{(\lambda, i) \in G \times V, \ (i, \lambda) \in T\}$.

Part III: Distributed sampling and reconstruction system

Recall: spatial signals $f = \sum_{i \in V} c_i \varphi_i$, where φ_i is the generating signal at innovative position $i \in V$, and amplitudes $c_i, i \in V$, are bounded.

Our distributed sampling/reconstruction system (DSRS) by the connected undirected graph

$$\mathcal{H} := (G \cup V, S \cup T \cup T^*).$$

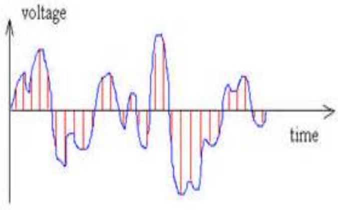


Part V: Sampling

- Consider spatial signals $f = \sum_{i \in V} c_i \varphi_i$, where φ_i is the generating signal at innovative position $i \in V$, and amplitudes $c_i, i \in V$, are bounded.[2]
- Then the sensing vector

$$\mathbf{y} = (\langle f, \psi_\lambda \rangle)_{\lambda \in G} \tag{1}$$

where ψ_λ is the impulse response of the sensing device located at position $\lambda \in G$.



[2] M. Vetterli, P. Marziliano, and T. Blu (2002); and Q. Sun (2008)

Part V: Sensing matrices for sampling

- The sensing matrix associated with our SDS is given by

$$\mathbf{S} = (\langle \psi_\lambda, \varphi_i \rangle)_{\lambda \in G, i \in V}, \tag{2}$$

  where ψ_λ reflects characteristic of the acquisition device at sensing location $\lambda \in G$, and φ_i is the generating signal at innovative position $i \in V$.
- sampling procedure: The sensing matrix $\mathbf{S}$ maps the amplitude vector $\mathbf{c} = (c_i)_{i \in V}$ of a signal $f = \sum_{i \in V} c_i \varphi_i$ into its sensing vector $\mathbf{y} = (\langle f, \psi_\lambda \rangle)_{\lambda \in G}$,

$$\mathbf{y} = \mathbf{S}\mathbf{c}. \tag{3}$$

Part V: Sensing matrices with off-diagonal decay

- Off-diagonal decay:
  Acquisition devices in our SDS has limited sensing ability and they could essentially catch signals not from their locations.

- Jaffard class $\mathcal{J}_\alpha(\mathcal{G}, \mathcal{V})$[3]:

$$\mathcal{J}_\alpha(\mathcal{G}, \mathcal{V}) := \Big\{(a(\lambda, i))_{\lambda\in G, i\in V}, \sup_{\lambda\in G, i\in V} (1+\rho_{\mathcal{H}}(\lambda, i))^\alpha |a(\lambda, i)|, \quad \alpha \ge 0\Big\}.$$

[3] S. Jaffard, *Ann. Inst. Henri Poincaré*, 7(1990), 461-476.

Part VI: Reconstruction in presence of bounded noise

Reconstructing a signal from sampling data in the presence of noises is a leading problem in sampling theory.

- How to reconstruct $\mathbf{c}$ approximately from noisy samples

$$\mathbf{z} = \mathbf{S}\mathbf{c} + \eta.$$

- Sampling error

$$\|\mathbf{S}\mathbf{c}_1 - \mathbf{S}\mathbf{c}_2\|_\infty \le C\|\mathbf{S}\|_{\mathcal{J}_\alpha(\mathcal{G},\mathcal{V})}\|\mathbf{c}_1 - \mathbf{c}_2\|_\infty \quad (4)$$

for all $\mathbf{c}_1, \mathbf{c}_2 \in \ell^\infty$.

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Part VI: Conventional minimization method

Consider the scenario that sampling data is corrupted by bounded noise η,

$$\mathbf{z} = \mathbf{S}\mathbf{c} + \boldsymbol{\eta}, \tag{5}$$

where $\mathbf{c}, \boldsymbol{\eta} \in \ell^\infty$.

- Conventional signal reconstruction:

$$\mathbf{d} := \operatorname{argmin}_{\mathbf{d}\in\ell^\infty} \|\mathbf{S}\mathbf{d} - \mathbf{z}\|_\infty. \tag{6}$$

- Expectation:

$$\|\mathbf{d} - \mathbf{c}\|_\infty \le C\|\boldsymbol{\eta}\|_\infty, \tag{7}$$

  where C is a positive constant.

How to meet our expectation?

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Part VI: Stability of sensing matrices

For $1 \le p \le \infty$, a matrix $\mathbf{A}$ is said to have ℓ^p-stability if

$$A\|\mathbf{c}\|_p \le \|\mathbf{Ac}\|_p \le B\|\mathbf{c}\|_p, \ \mathbf{c} \in \ell^p. \tag{8}$$

Meet the expectation if the sensing matrix $\mathbf{S}$ of our SDS has ℓ^∞-stability:

Proof. Given noise data $\mathbf{z} = \mathbf{Sc} + \eta$, corrupted by bounded noise η, do minimization $\mathbf{d} := \mathrm{argmin}_{\mathbf{d}\in\ell^\infty} \|\mathbf{Sd} - \mathbf{z}\|_\infty$. Then

$$\|\mathbf{Sd} - \mathbf{Sc} - \boldsymbol{\eta}\|_\infty = \|\mathbf{Sd} - \mathbf{z}\|_\infty \le \|\mathbf{Sc} - \mathbf{z}\|_\infty = \|\boldsymbol{\eta}\|_\infty,$$

which implies that

$$\|\mathbf{Sd} - \mathbf{Sc}\|_\infty \le 2\|\boldsymbol{\eta}\|_\infty.$$

Our main mathematical contributions

- How to verify ℓ^∞-stability.
- How to solve ℓ^∞ minimization

- Our first topic: Verification ℓ^∞-stability $\Leftarrow$ ℓ^2-stability (positive eigenvalues of $\mathbf{S}^T\mathbf{S}$) $\Leftarrow$ local stability criterion
- Our second topic: ℓ^∞ minimization (global linear programming) $\Leftarrow$ least square solution $\tilde{\mathbf{d}}$ is sub-optimal

$$\|\tilde{\mathbf{d}} - \mathbf{c}\|_\infty \leq C\|\eta\|_\infty. \tag{9}$$

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Part VII: From ℓ^2-stability to ℓ^∞-stability

Theorem 1 (Cheng, Jiang and S.)

Assume that $\mathbf{S} \in \mathcal{J}_\alpha(\mathcal{G}, \mathcal{V})$ *for some* $\alpha > d$. *If* $\mathbf{S}$ *has* ℓ^2*-stability, then it has* ℓ^p*-stability for all* $1 \le p \le \infty$.

By the Theorem 1, the ℓ^∞-stability of a matrix $\mathbf{A}$ in $\mathcal{J}_\alpha(\mathcal{G}, \mathcal{V})$ reduces to its ℓ^2-stability.

How to verify the ℓ^2-stability?

Part VII: Local criterion for stability of sensing matrices

The ℓ^2-stability of a sensing matrix $\mathbf{S}$ is equivalent to positive eigenvalues of $\mathbf{S}^T\mathbf{S}$. But it is not practical, as eigenvalue of $\mathbf{S}^T\mathbf{S}$ cannot be evaluated by the SDS itself.

For $\lambda \in G$ and positive integer N, define

$$\chi_\lambda^N : \ (c(i))_{i\in V} \longmapsto \big(c(i)\chi_{B_{\mathcal{H}}(\lambda,N)\cap V}(i)\big)_{i\in V}$$

Theorem 2 (Cheng, Jiang and S.)

Let $\mathbf{S} \in \mathcal{J}_\alpha(\mathcal{G}, \mathcal{V}), \alpha > d$. The it has ℓ^2-stability if and only if its quasi-main submatrices $\chi_\lambda^{2N} \mathbf{A} \chi_\lambda^N, \lambda \in G$, of size $O(N^d)$ have uniform ℓ^2-stability for large (but fixed) N,

$$\|\chi_\lambda^{2N} \mathbf{A} \chi_\lambda^N \mathbf{c}\|_2 \ge A \|\mathbf{A}\|_{\mathcal{J}_\alpha(\mathcal{G},\mathcal{V})} \|\chi_\lambda^N \mathbf{c}\|_2, \ \mathbf{c} \in \ell^2(V), \qquad (10)$$

for all $\lambda \in G$.

Part VIII: Least squares

Recall: $\mathbf{z} = \mathbf{S}\mathbf{c} + \boldsymbol{\eta}$.
The "least squares" solution

$$\tilde{\mathbf{d}} = (\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T\mathbf{z}$$

of the linear system $\mathbf{A}\mathbf{d} = \mathbf{z}$ is well-defined if $\mathbf{S} \in \mathcal{J}_\alpha(\mathcal{G}, \mathcal{V})$ for some $\alpha > d$ and it has ℓ^2-stability. Moreover, it is a suboptimal reconstruction.

Theorem 3 (Cheng, Jiang and S.)

There exists a positive constant C such that

$$\|\tilde{\mathbf{d}} - \mathbf{c}\|_\infty \leq C\|\boldsymbol{\eta}\|_\infty. \tag{11}$$

Part VIII: ℓ^∞-minimization and ℓ^2-minimization

Conventional minimization:

$$\mathbf{d} := \mathrm{argmin}_{\mathbf{d}\in\ell^\infty} \|\mathbf{S}\mathbf{d}-\mathbf{z}\|_\infty.$$

"least squares" solution

$$\tilde{\mathbf{d}} = (\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T\mathbf{z} = \mathrm{argmin}_{\mathbf{d}\in\ell^2} \|\mathbf{S}\mathbf{d}-\mathbf{z}\|_2$$

Comparison:

- The "least square" solution $\tilde{\mathbf{d}}$ has explicit expression and it depends on the noisy sampling data $\mathbf{z}$ linearly,
- The optimal solution $\mathbf{d}$ does not have a closed form, and it depends on $\mathbf{z}$ nonlinearly. (global linear programming)

Part IX: Spatially distributed algorithm I

Question: How to find least squares $\tilde{\mathbf{d}} = (\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T\mathbf{z}$?

Let $\mathbf{c} \in \ell^\infty(V)$ and $\mathbf{y} = \mathbf{S}\mathbf{c}$ be sampling data vector. Set initial $\mathbf{c}_0 = 0$ and $\mathbf{y}_0 = \mathbf{y}$, and define $\mathbf{c}_n, \mathbf{y}_n, n \ge 1$, iteratively by

$$\begin{cases} \mathbf{z}_{n;\lambda,N} = \mathbf{R}_{\lambda,N}\mathbf{y}_n, \ \ \lambda \in G, \\ \mathbf{z}_n = \dfrac{\sum_{\lambda\in G}\chi_\lambda^N \mathbf{z}_{n;\lambda,N}}{\sum_{\lambda\in G}\chi_{B(\lambda,N)}}, \\ \mathbf{c}_{n+1} = \mathbf{c}_n + \mathbf{z}_n, \\ \mathbf{y}_{n+1} = \mathbf{y}_n - \mathbf{S}\mathbf{z}_n, \end{cases} \tag{12}$$

where

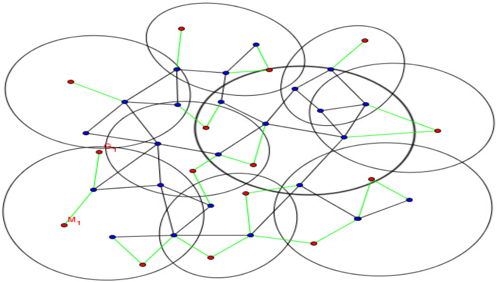
$$\mathbf{R}_{\lambda,N} = (\chi_\lambda^{2N}\mathbf{S}^T\chi_\lambda^{4N}\mathbf{S}\chi_\lambda^{2N})^{-1}\chi_\lambda^{2N}\mathbf{S}^T\chi_\lambda^{4N}.$$

Part IX: Spatially distributed algorithm II

The implementation of the algorithm (12) can be distributed for devices in an SDS in each iteration.

$$\mathbf{z}_{n;\lambda,N} = \mathbf{R}_{\lambda,N}\mathbf{y}_n, \ \lambda \in G$$

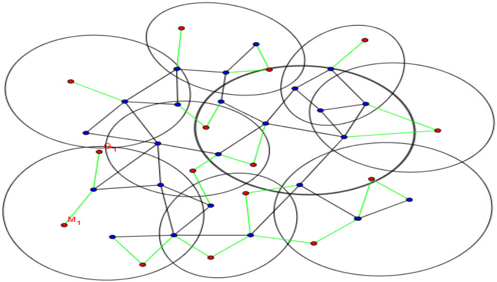
(i) For the device located at $\lambda \in G$, first get data $\mathbf{y}_n(\gamma)$ from neighboring devices located at $\gamma \in B(\lambda, 4N)$, and then generate local correction $\mathbf{z}_{n,\lambda,N}$;



Part IX: Spatially distributed algorithm III

$$\mathbf{z}_n = \frac{\sum_{\lambda \in G} \chi_\lambda^N \mathbf{z}_{n;\lambda,N}}{\sum_{\lambda \in G} \chi_{B(\lambda,N)}}$$

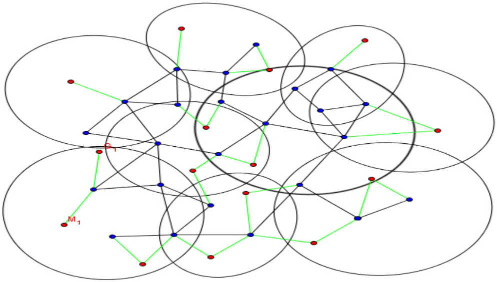
(ii) Then send the correction $\mathbf{z}_{n,\lambda,N}$ to neighboring devices located at $\gamma \in B(\lambda, N)$ and compute the correction $\mathbf{z}_n$.



Part IX: Spatially distributed algorithm IV

$$\mathbf{c}_{n+1} = \mathbf{c}_n + \mathbf{z}_n$$

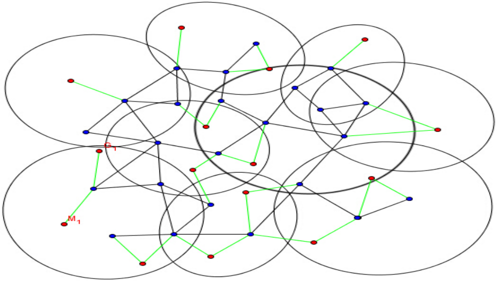
(iii) Next add the correction $\mathbf{z}_n$ to old prediction $\mathbf{c}_n$ to create new prediction $\mathbf{c}_{n+1}$.



Part IX: Spatially distributed algorithm V

$$\mathbf{y}_{n+1} = \mathbf{y}_n - \mathbf{S}\mathbf{z}_n$$

(iv) Send $\mathbf{z}_n(\lambda)$ to neighboring devices at $\gamma \in B(\lambda, M)$ and compute new correction $\mathbf{y}_{n+1}$ on sampling data, where M is the bandwidth of the sensing matrix $\mathbf{S}$.



Part IX: Exponential Convergence of our distributed algorithm

The complexity of the above distributed algorithm (using data processing and communication capabilities).

- Storage requirement $O(N^d)$
- Computing power $O(N^d)$ for each iteration
- Communication cost $O(N^{d+\beta})$ if the communication cost between two devices λ, λ' is propositional to $\rho(\lambda, \lambda'))^\beta$ (usually $\beta = 1$)
- Number of iteration $\frac{\ln(1/\epsilon)}{\ln N}$, where ϵ is accuracy requirement.

Theorem 4 (Cheng, Jiang and S.)

If an SDS has ℓ^2-stability. Then for large N, the sequence $\mathbf{c}_n, n \geq 0$, converges to $\mathbf{c}$ exponentially,

$$\|\mathbf{c}_n - \mathbf{c}\|_\infty \leq r_1^n \|\mathbf{c}\|_\infty. \tag{13}$$

Part X: Simulations

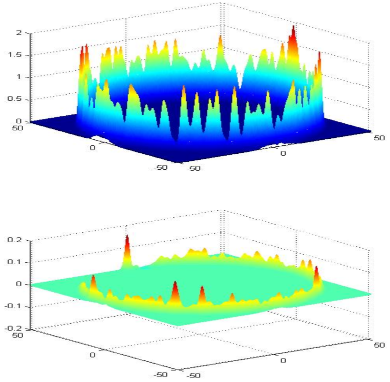
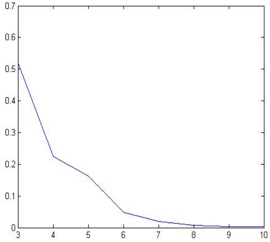
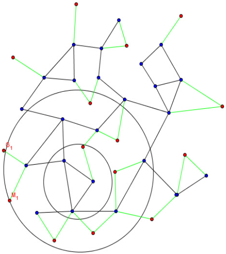


Figure: Top is the signal $f = \sum_{1\le l\le L} c_l\phi_l$ with 2-D Gaussian kernel; Bottom is the difference of the original signal and the reconstructed signal using the algorithm with $N = 4$.

Part IX: faster convergence

- Minimal storage, computing, and communications capabilities by selecting smaller integer *N*.
- Fast convergence (less delay for reconstruction) by selecting large *N* when large storage and better computing and communications capabilities available.
- For sufficiently large *N*, no iteration necessary, cf. finite-section method.





Math and Philosophy

- Philosophy: *Think global, act local*: Local criterion for stability, distributed algorithms
- Mathematics: Property of matrices (operators, functions) by preserved by certain mapping, such as inverse.
  1. Wiener's lemma: If A has certain off-diagonal decay and it is invertible, then A^{-1} has the same off-diagonal decay. For instance, if $A = (a(m,n))_{m,n\in\mathbf{Z}}$ has polynomial off-diagonal decay,
  $$|a(m,n)| \le C(1+|m-n|)^{-\alpha}, \quad m,n\in\mathbf{Z}$$
  and A has bounded inverse, then A^{-1} has the same off-diagonal decay.
  2. $A \longmapsto A^{-1}$ (Inverse-closed algebra). [4]

[4] **I. Gohberg, M. A. Kaashoek, and H. J. Woerdeman**, 1989; **S. Jaffard**, 1990; **J. Sjöstrand**, 1994; **Shin and Sun**, 2009. **K. Gröchenig and M. Leinert**, 2011; **S.**, 2007, 2009; **N. Motee and Q. Sun**, Siam Optimization 2017.

Summary

- Framework for spatially distributed system for signal and sampling.
- Stability of sensing matrices.
- Distributed algorithm for fast reconstruction
- Distributed algorithms for global optimization

$$\mathrm{argmin}\ \|Ax - b\|_2^2 + \lambda \|Bx\|_1.$$

- Philosophy: *Think global, act local*; Mathematics: Property of matrices (operators, functions) by preserved by certain mapping, such as inverse.

Thank You!