Proximal alternating algorithms in dictionary learning

Chenglong Bao

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ABSTRACT

In recent years, sparse coding has been widely used in many applications ranging from image processing to pattern recognition. Most existing sparse coding based applications require solving a class of challenging non-smooth and non-convex optimization problems. In this talk, I will review some proximal alternating algorithms for solving such problem and give rigorous convergence analysis. Experiments show that the proposed method achieves similar results with less computation when compared to widely used methods such as K-SVD.

Non-Convex Methods for Low-Rank Matrix Reconstruction

Jian-Feng Cai

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ABSTRACT

We present a framework of non-convex methods for reconstructing a low rank matrix from its limited information, which arises from numerous practical applications in machine learning, imaging, signal processing, computer vision, etc. Our methods will be applied to several concrete example problems such as matrix completion, phase retrieval, and spectral compressed sensing with super resolution. We will also provide theoretical guarantee of our methods for the convergence to the correct low-rank matrix.

Wavelet frames on the sphere for sparse representations in high angular resolution diffusion imaging

Weiqiang Chen

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ABSTRACT

Over the past two decades, wavelet frames are widely used in image and signal processing applications as they yield redundant and flexible data representations. This talk describes how wavelet frames defined on the sphere can be constructed to sparsely represent signals for High Angular Resolution Diffusion Imaging (HARDI), a relatively recent non-invasive brain imaging technique. Using carefully weighted linear combinations of (modified) spherical harmonics involving Gaussian quadrature-like weights and nodes, wavelet frames are constructed for the space of symmetric, square-integrable functions on the unit sphere, in order to represent HARDI signals. These wavelet frames can then be applied to denoise HARDI signals, with tight framelet filters also used to impose spatial regularization for enhanced denoising performances. Our experiments suggest that the proposed wavelet frame-based approach generally denoises highly corrupted HARDI signals more cost-effectively than the conventional spherical harmonics-based and spherical ridgelets-based approaches.

Sparse Approximation: From Image Restoration to High Dimensional Classification

Bin Dong

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ABSTRACT

The first half of my talk reviews some of our work on sparse approximation in image restoration. In a series of papers, we established rigorous connections between wavelet frame transforms and differential operators in variational framework, as well as for nonlinear evolution PDEs. Such connections not only provide us with new and fascinating insights on both wavelet frame and differential operator based approaches for image restoration, but also enable us to introduce new models and algorithms that combine the merits of both approaches. In the second half of my talk, I will discuss how these findings from image restoration can further guide us in the process and analysis of more general data sets in high dimensional spaces.

Signal Modeling: From Convolutional Sparse Coding to Convolutional Neural Networks

Michael Elad

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ABSTRACT

Within the wide field of sparse approximation, convolutional sparse coding (CSC) has gained increasing attention in recent years. This model assumes a structured-dictionary built as a union of banded Circulant matrices. Most of the attention has been devoted to the practical side of CSC, proposing efficient algorithms for the pursuit problem, and identifying applications that benefit from this model. Interestingly, a systematic theoretical understanding of CSC seems to have been left aside, with the assumption that the existing classical results are sufficient.

In this talk we start by presenting a novel analysis of the CSC model and its associated pursuit. Our study is based on the observation that while being global, this model can be characterized and analyzed locally. We show that uniqueness of the representation, its stability with respect to noise, and successful greedy or convex recovery are all guaranteed assuming that the underlying representation is locally sparse. These new results are much stronger and informative, compared to those obtained by deploying the classical sparse theory.

Armed with these new insights, we proceed by proposing a multi-layer extension of this model, ML-CSC, in which signals are assumed to emerge from a cascade of CSC layers. This, in turn, is shown to be tightly connected to Convolutional Neural Networks (CNN), so much so that the forward-pass of the CNN is in fact the Thresholding pursuit serving the ML-CSC model. This connection brings a fresh view to CNN, as we are able to attribute to this architecture theoretical claims such as uniqueness of the representations throughout the network, and their stable estimation, all guaranteed under simple local sparsity conditions. Lastly, identifying the weaknesses in the above scheme, we propose an alternative to the forward-pass algorithm, which is both tightly connected to deconvolutional and recurrent neural networks, and has better theoretical guarantees.

This talk summarizes a research project led by three of my PhD students, Vardan Papyan, Jeremias Sulam, and Yaniv Romano, and the detailed description is reported in [1, 2].

References

[1] V. Papyan, J. Sulam, and M. Elad *Working Locally Thinking Globally: Theoretical Guarantees for Convolutional Sparse Coding,*, submitted to IEEE Trans. on Signal Processing.

[2] V. Papyan, Y. Romano, and M. Elad, *Convolutional Neural Networks Analyzed via Convolutional Sparse Coding*, submitted to the Journal of Machine Learning Research.

What happened to the Representations of Perception?

Cornelia Fermüller

University of Maryland, USA

ABSTRACT

Currently, mainstream approaches to Vision relate symbolic information directly to the visual input. However, the vision of active agents employs intermediate representations essential for the perception-action cycle supporting the agent´s actions. I will describe approaches and ideas on the implementations of such representations, with a focus on action understanding. First, we developed so-called mid-level grouping mechanisms implemented as image operators to obtain objects in images and image depth data through attention, segmentation and recognition processes. Second, the mid-level processes serve as the interface between image processing and cognition, and to realize top-down modulation, we studied the implementation of feedback loops in neural networks and dimensionality reduction in its intermediate layers. Third, the representations of action contain not only vision but also motoric information. Our approach implements the fusion of different information sources in neural networks.

The unitary extension principle on locally compact abelian groups

Say Song Goh

National University of Singapore, Singapore

ABSTRACT

The unitary extension principle (UEP) by Ron and Shen provides an elegant method for the construction of a multi-generated tight wavelet frame on $\mathbb{R}^s$ based on a given refinable function. We shall present a generalization of the UEP to locally compact abelian groups. The generalization covers both the stationary and the nonstationary case, which include among others the classical UEP as well as its subsequent variants for the nonstationary case on $\mathbb{R}^s$ and the periodic case. In our general setting, the conditions in the UEP are formulated in terms of the annihilators of nesting lattices in the group and a matrix extension condition, involving refinement and wavelet filters, on the fundamental domains associated with these annihilators. The resulting frames on the dual group are generated by modulates of a collection of functions, which correspond, via the Fourier transform, to a generalized shift-invariant system on the group. We shall also provide general constructions, based on B-splines on the group itself as well as on characteristic functions on the dual group. This is joint work with Ole Christensen.

Multivariate wavelet frames through constant matrix completion via the duality principle

ZHITAO FAN[a], ANDREAS HEINECKE[a,b], HUI JI[a], AND ZUOWEI SHEN[a]

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ABSTRACT

The duality principle, ultimately a statement about adjoint operators, is a universal principle in frame theory. We take a broad perspective on the duality principle and discuss how the mixed unitary extension principle for MRA-wavelet frames can be viewed as the duality principle in a sequence space. This leads to a construction scheme for dual MRA-wavelet frames which is strikingly simple in the sense that it only requires the completion of an invertible constant matrix. Under minimal conditions on the multiresolution analysis our construction guarantees the existence and easy constructability of multivariate non-separable dual MRA-wavelet frames of compactly supported wavelets.

References

[1] Fan Z., Heinecke A., Shen Z., *Duality for frames*, Journal of Fourier Analysis and Applications, pp. 71–136, 2016.

[2] Fan Z., Ji H., Shen Z., *Dual Gramian analysis: duality principle and unitary extension principle*, Mathematics of Computation, pp. 239–270, 2016.

Fast Computing via Recursive Dyadic Partitioning for Statistical Dependency

Xiaoming Huo

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ABSTRACT

Distance correlation had been introduced as a better alternative to the celebrated Pearson's correlation. Distance correlation is one of many distance-based methods in statistics, whose key idea is to develop a statistic that is based on pairwise distances among observations. The existing algorithm for the distance correlation seemingly requires an $\mathrm{O}(n^2)$ algorithm, and I will show how it can be done in $O(n \log n)$. The essence of the fast algorithm is a recursive dyadic partition of the search space, with a corresponding faster dynamic programming algorithm. Because of this approach, many other statistical dependency related quantities can be computed efficiently. I will give some other examples.

Toward spatially-varying blind image deconvolution: models and techniques

Hui Ji

National University of Singapore, Singapore

ABSTRACT

Blind Image de-convolution is one challenging inverse problem with many applications in practice. Blind deconvolution aims at recovering the clear image from its blurred observation without knowing how it is blurred. In many realistic scenarios, the blurring process is non-stationary in the sense that different image regions are blurred by different kernels, which makes it even more difficult. In this talk, I will present several mathematical models and techniques toward solving non-stationary blind image deblurring problems arising from industrial imaging and digital photography, including spatially-varying blind motion deblurring and defocus map estimation for out-of-focus blurring.

Artificial Intelligence in Medicine (AIM)

Steve Jiang

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ABSTRACT

Recently we have seen huge advancements in artificial intelligence (AI) mainly due to advancements in algorithms (better algorithms like deep reinforcement learning), computing power (more powerful hardware based on GPU or more dedicated platforms like Alphabet's tensor processing unit (TPU), Qualcomm's neural processing unit (NPU), Nvidia's deep learning chip, or IBM's TrueNorth neuromorphic computing platform), big data (more readily available large datasets), and cloud computing. A milestone in this field is AlphaGo's historic victory last year against the best Go players in the world. AI has been identified as the next big area of innovation and attracted tremendous amount of talent and capital. One major application for AI is healthcare. IBM Watson is a good example that has outperformed human doctors in some medical areas such as breast cancer diagnosis from mammography. In this talk we will review the current status and our perspective of artificial intelligence in medicine (AIM). Specifically we will discuss where we are, what is the future, and what are challenges. The goal is to identify potential areas for collaborations between mathematicians and clinicians to further advance AIM.

Understanding Data from Incomplete Inter-Point Distance via Locally Low-rank Matrix Completion and Geometric PDEs

RONGJIE LAI

Rensselaer Polytechnic Institute, USA

ABSTRACT

The problem of global understanding of point clouds represented as incomplete inter-point distance has many applications in 3D modeling, sensor network localization as well as protein structuring. Without considering time-consuming global coordinates reconstruction, we propose to only reconstruct manifold locally using low-rank matrix completion theory and to conduct global understanding using geometric PDEs to link local information and global information. I will demonstrate efficiency and effectiveness of our method. If time permits, I may also discuss an extension of this idea of using manifold based locally low-rank matrix completion to some image restoration problems.

Minimizing the Difference of L_1 and L_2 norms with Applications

Yifei Lou

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ABSTRACT

A fundamental problem in compressive sensing (CS) is to reconstruct a sparse signal under a few linear measurements far less than the physical dimension of the signal. Currently, CS favors incoherent systems, in which any two measurements are as little correlated as possible. In reality, however, many problems are coherent, in which case conventional methods, such as L_1 minimization, do not work well. In this talk, I will present a novel non-convex approach, which is to minimize the difference of L_1 and L_2 norms (L_1-L_2) in order to promote sparsity. In addition to theoretical aspects of the L_1-L_2 approach, I will discuss two minimization algorithms. One is the difference of convex (DC) function methodology, and the other is based on a proximal operator, which makes some L_1 algorithms (e.g. ADMM) applicable for L_1-L_2. Experiments demonstrate that L_1-L_2 improves L_1 consistently and it outperforms L_p $(0 < p < 1)$ for highly coherent matrices. Some applications will be discussed, including super-resolution, machine learning, and hyperspectral unmixing.

Multiscale representation of deformation via Beltrami coefficients

Lok Ming Lui

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ABSTRACT

Analyzing the deformation pattern of an object is crucial in various fields, such as in computer visions and medical imaging. A deformation can be considered as a combination of local and global deformations at different locations. To fully understand and analyze the deformation pattern, extracting deformation components of various scales and locations is necessary. We propose an algorithm for the multi-scale decomposition of a bijective deformation using quasi-conformal theories. A deformation of an object can be described as a orientation-preserving homeomorphism of a two dimensional domain. The mapping is then represented by its associated Beltrami coefficient (**BC**), which measures the local geometric (conformality) distortion of the deformation. The **BC** is a complex-valued function defined on the source domain. By applying the wavelet transform on the **BC**, the **BC** can be decomposed into different components of different frequencies compactly supported in different sub-domains. Quasi-conformal mappings associated to different components of the **BC** can be reconstructed by solving the Beltrami's equation. A multi-scale decomposition of the deformation can then be constructed. To validate our proposed algorithm, we test it on synthetic examples as well as real medical data. Experimental results show the efficacy of our proposed model to decompose deformations at multiple scales and locations.

Overcoming the Curse of Dimensionality for Hamilton-Jacobi equations with Applications to Control and Differential Games

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ABSTRACT

It is well known that certain Hamilton-Jacobi partial differential equations (HJ PDE´s) play an important role in analyzing control theory and differential games. The cost of standard numerical algorithms for HJ PDE´s is exponential in the space dimension and time, with huge memory requirements. Here we propose and test methods for solving a large class of these problems without the use of grids or significant numerical approximation. We begin with the classical Hopf and Hopf-Lax formulas which enable us to solve state independent problems via variational methods originating in compressive sensing with remarkable results. We can evaluate the solution in 10^{-4} to 10^{-8} seconds per evaluation on a laptop. The method is embarrassingly parallel and has low memory requirements.

Recently, with a slightly more complicated, but still embarrassingly parallel method, we have extended this in great generality to state dependent HJ equations, apparently, with the help of parallel computers, overcoming the curse of dimensionality for these problems. The term, ǆż″curse of dimensionalityǆż″ was coined by Richard Bellman in 1957 when he did his classic work on dynamic optimization

Multiscale High-Dimensional Learning and Deep Neural Networks

Stéphane Mallat

École Normale Supérieure, France

ABSTRACT

Data classification and regression typically require to approximate high-dmensional functions. Avoiding the curse of dimensionality raises issues in many branches of mathematics including statistics, probability, harmonic analysis and geometry. Recently, deep convolutional networks have obtained spectacular results for image understanding, audio recognition,and all kind of data analysis problems.

We shall review their architecture, and introduce a scattering model implemented with multiscale wavelet filters. It provides sparse representations, and outputs invariants relatively to symmetry groups of classification and regression functions. Unsupervised learning applications will be shown to generate non-Gaussian random processes including image and audio textures, Ising models and turbulent fluids. Supervised learning applications to image and audio classification will also be shown. Learning complex physical properties will be demonstrated through computations of quantum molecular energies, with no prior quantum physics knowledge.

What mathematical algorithms can do for the real (and even fake) world

Stanley Osher

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ABSTRACT

I will give a very personal overview of the evolution of mainstream applied mathematics from the early 60's onwards. This era started pre computer with mostly analytic techniques, followed by linear stability analysis for finite difference approximations, to shock waves, to image processing, to the motion of fronts and interfaces, to compressive sensing and the associated optimization challenges, to the use of sparsity in Schrodinger's equation and other PDE's, to overcoming the curse of dimensionality in parts of control theory and in solving the associated high dimensional Hamilton-Jacobi equations

Manifold learning for brain morphological shapes

Anqi Qiu

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ABSTRACT

We present the algorithm, Locally Linear Diffeomorphic Metric Embedding (LLDME), for constructing efficient and compact representations of surface-based brain shapes whose variations are characterized using Large Deformation Diffeomorphic Metric Mapping (LDDMM). Our hypothesis is that the shape variations in the infinite-dimensional diffeomorphic metric space can be captured by a low-dimensional space. To do so, traditional Locally Linear Embedding (LLE) that reconstructs a data point from its neighbors in Euclidean space is extended to LLDME that requires interpolating a shape from its neighbors in the infinite-dimensional diffeomorphic metric space. This is made possible through the conservation law of momentum derived from LDDMM. It indicates that initial momentum a linear transformation of the initial velocity of diffeomorphic flows, at a fixed template shape determines the geodesic connecting the template to a subject's shape in the diffeomorphic metric space and becomes the shape signature of an individual subject. This leads to the compact linear representation of the nonlinear diffeomorphisms in terms of the initial momentum. Since the initial momentum is in a linear space, a shape can be approximated by a linear combination of its neighbors in the diffeomorphic metric space. In addition, we provide efficient computations for the metric distance between two shapes through the first order approximation of the geodesic using the initial momentum as well as for the reconstruction of a shape given its low-dimensional Euclidean coordinates using the geodesic shooting with the initial momentum as the initial condition. Experiments are performed on the hippocampal shapes of 302 normal subjects across the whole life span (18 94 years). Compared with Principal Component Analysis and ISOMAP, LLDME provides the most compact and efficient representation of the age-related hippocampal shape. Even though the hippocampal volumes among young adults are as variable as those in older adults, LLDME disentangles the hippocampal local shape variation from the hippocampal size and thus reveals the nonlinear relationship of the hippocampal morphometry with age.

An Approach to Statistical Shape Analysis

Fadil Santosa

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ABSTRACT

In statistical shape analysis the goal is to obtain characteristics such as mean, standard deviation, etc., from a set of shapes. While much progress in this area has occurred in the past four decades, many challenges remain. This presentation will review several of the important developments in this field. An approach based on Fourier analysis is proposed and its capabilities demonstrated.

Image Restoration and Beyond

Zuowei Shen

National University of Singapore, Singapore

ABSTRACT

We are living in the era of big data. The discovery, interpretation and usage of the information, knowledge and resources hidden in all sorts of data to benefit human beings and to improve everyone's day to day life is a challenge to all of us. The huge amount of data we collect nowadays is so complicated, and yet what we expect from it is so much. This provides many challenges and opportunities to many fields. As images are one of the most useful and commonly used types of data, in this talk, we start from reviewing the development of the wavelet frame (or more general redundant system) based approach for image restoration. We will observe that a good system for any data, including images, should be capable of effectively capturing both global patterns and local features. One of the examples of such system is the wavelet frame. We will then show how models and algorithms of wavelet frame based image restoration are developed via the generic knowledge of images. Then, the specific information of a given image can be used to further improve the models and algorithms. Through this process, we shall reveal some insights and understandings of the wavelet frame based approach for image restoration and its connections to other approaches, e.g. the partial differential equation based methods. Finally, we will also show, by many examples, that ideas given here can go beyond image restoration and can be used to many other applications in data science.

Spatially Distributed Systems for Signal Sampling and Reconstruction

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[c] *University of Central Florida, USA*

ABSTRACT

A spatially distributed network contains a large amount of agents with limited sensing, data processing, and communication capabilities. Recent technological advances have opened up possibilities to deploy spatially distributed networks for signal sampling and reconstruction. In this talk, we will introduce a graph structure for a distributed sampling and reconstruction system by coupling agents in a spatially distributed network with innovative positions of signals.

A fundamental problem in sampling theory is the robustness of signal reconstruction in the presence of sampling noises. For a distributed sampling and reconstruction system, the robustness could be reduced to the stability of its sensing matrix. In a traditional centralized sampling and reconstruction system, the stability of the sensing matrix could be verified by its central processor, but the above procedure is infeasible in a distributed sampling and reconstruction system as it is decentralized. In this talk, I will talk about stability criterion that could be verified in a distribute manner.

In this talk, we also discuss an exponentially convergent distributed algorithm for signal reconstruction, that provides a suboptimal approximation to the original signal in the presence of bounded sampling noises.

Sketchy decisions: Low-rank matrix optimization with optimal storage

Joel Tropp

California Institute of Technology, USA

ABSTRACT

Convex matrix optimization problems with low-rank solutions play a fundamental role in signal processing, statistics, and related disciplines. These problems are difficult to solve because of the cost of maintaining the matrix decision variable, even though the low-rank solution has few degrees of freedom. This talk presents the first algorithm that provably solves these problems using optimal storage. The algorithm produces high-quality solutions to large problem instances that, previously, were intractable.

Joint with Volkan Cevher, Roarke Horstmeyer, Quoc Tran-Dinh, Madeleine Udell, and Alp Yurtsever.

Dual principal component pursuit

Manolis Tsakiris and René Vidal

Johns Hopkins University, USA

ABSTRACT

State-of-the-art methods for robust subspace learning and clustering are based on sparse and low-rank representation theory and convex optimization algorithms. Existing theoretical results for guaranteeing the correctness of such methods require the dimensions of the subspaces to be small relative to the dimension of the ambient space. When this assumption is violated, as is, e.g., in the case of hyperplanes, existing methods are either computationally too intensive or lack sufficient theoretical support.

We propose a new approach to robust subspace learning and clustering called *Dual Principal Component Pursuit*. The proposed approach works with a dual representation of the subspaces and hence aims to find their orthogonal complement. We pose this problem as an ℓ_1-minimization problem on the sphere and show that, under certain conditions on the distribution of the data, any global minimizer of this non-convex problem gives a vector orthogonal to one of the subspaces. Moreover, we show that a global minimizer to the non-convex problem can be found by solving a sequence of linear programs. Experiments on synthetic and real data show that the proposed approach outperforms state-of-the art methods, especially in the case of subspaces of high relative dimension.

References

[1] Tsakiris M. and Vidal R. *Dual principal component pursuit*, ICCV Workshop on Robust Subspace Learning and Computer Vision, Santiago de Chile, 2015.

[2] Tsakiris M. and Vidal R. *Hyperplane clustering via dual principal component pursuit*, International Conference on Machine Learning, Sydney, 2017.

Multiresolution Analysis and Wavelets on Hierarchical Data Trees

Jianzhong Wang

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ABSTRACT

Construction of wavelets on a data graph has attracted great interests in data science. Several authors take the spectral approach to construction of wavelets on the graph. [1, 2]. However, the spectral approach do not provide the local information of data. It has limitation in many applications that treat data in spatial domain. In this presentation, we construct wavelet basis and framelets in the spatial domains. The earlier work in this aspect can be found in [3]. Our method adopt the following strategy: We first build a hierarchic data tree for a given data graph and then make a Multi-Resolution Analysis on the tree. Then we construct the wavelet basis and tight frames on each brunch of the tree. Finally, we integrate all of the brunch bases or framelets into the wavelet basis or framelets for the whole data set. We point out that, under the Hölder smoothness assumption on the function on the data, the coefficients of the wavelet decomposition of the function have the exponential decay. Thus, the classical wavelet analysis technique and tools can be applied to the wavelets on the data too.

References

[1] R. R. Coifman and M. Maggioni, *Diffusion wavelets in Special Issue on Diffusion Maps and Wavelets*, Appl. Comput. Harmon. Anal., **21** pp. 53–94, 2006.

[2] D.K. Hammond, P. Vndergheynst, and R. Gribonval, *Wavelets on graphs via spectral graph theory* Appl. Comput. Harmon. Anal., **30** pp. 129–150, 2011.

[3] M. Gavish, B. Nadler, and R. R. Coifman, *Multiscale wavelets on trees, graphs and high dimensional data: Theory and applications to semi supervised learning* in Proceedings of the 27th International Conference on machine Learning, Eds. Johannes Fürnkranz and Thorsten Joachims, pp. 367–374, 2010.

Data Recovery on Manifolds: A Theoretical Framework

Yang Wang

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ABSTRACT

Recovering data from compressed number of measurements is ubiquitous in applications today. Among the best know examples are compressed sensing and low rank matrix recovery. To some extend phase retrieval is another example. The general setup is that we would like to recover a data point lying on some manifold having a much lower dimension than the ambient dimension, and we are given a set of linear measurements. The number of measurements is typically much smaller than the ambient dimension. So the questions become: Under what conditions can we recover the data point from these linear measurements? If so, how? The problem has links to classic algebraic geometry as well as some classical problems on the embedding of projective spaces into Euclidean spaces and nonsingular bilinear forms. In this talk I'll give a brief overview and discuss some of the recent progresses.

Principal Sub-manifolds

ZHIGANG YAO

National University of Singapore, Singapore

ABSTRACT

We revisit the problem of finding principal components to the multivariate datasets, that lie on an embedded nonlinear Riemannian manifold within the higher-dimensional space. Our aim is to extend the geometric interpretation of PCA, while being able to capture the non-geodesic form of variation in the data. We introduce the concept of a principal sub-manifold, a manifold passing through the center of the data, and at any point of the manifold, it moves in the direction of the highest curvature in the space spanned by the eigenvectors of the local tangent space PCA. Compared to the recent work in the case where the sub-manifold is of dimension one (Panaretos et al. 2014), essentially a curve lying on the manifold attempting to capture the one-dimensional variation, the current setting is much more general. The principal sub-manifold is therefore an extension of the principal flow, accommodating to capture the higher dimensional variation in the data. We show the principal sub-manifold yields the usual principal components in Euclidean space. By means of examples, we illustrate that how to find, use and interpret principal sub-manifold with an extension of using it in shape analysis. (This is a joint work with Tung Pham)

References

[1] V. M. Panaretos, T. Pham and Z. Yao Principal flows, *Journal of the American Statistical Association*, 109, pp. 424–436, 2014.

Convex and Non-Convex Optimization in Image Recovery and Segmentation

Tieyong Zeng

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ABSTRACT

In this talk, we present some recent progress on variational approaches for image recovery and segmentation. First, a new convex variational model for restoring images degraded by blur and Rician noise is proposed. Based on the statistical property of the noise, a quadratic penalty function technique is utilized to obtain a strictly convex model under mild condition, which ensures the uniqueness of the solution and the stabilization of the algorithm. Numerical results are presented to demonstrate the good performance of our approach. The idea of convex relaxation is then extended to other image recovery and segmentation tasks. Finally, we also discuss the image recovery issue in the framework of dictionary learning if time permitted.

Bi-modality joint image reconstruction and fusion by tight frame

Xiaoqun Zhang

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ABSTRACT

Recent technical advances lead to the coupling of multi-modality for joint image reconstruction, information fusion and analysis. For instance, PET-MRI scanners enable to acquire functional and anatomical data simultaneously. For the joint reconstruction problem, the goal is to improve the reconstruction quality of one modality by utilizing complementary information from the other modality. We consider a wavelet tight frame based PET-MRI joint reconstruction model via the joint sparsity of tight frame coefficients. A non-convex balanced approach is adopted to take the different sparsity patterns of PET-MRI images into account. The proposed nonconvex and nonsmooth model is solved by a proximal alternating minimization algorithm and the global convergence is established. The numerical experiments show that the our proposed models achieve better performance over the existing PET-MRI joint reconstruction models.

The second model that we consider is for PET and MRI image fusion. The purpose is to combine complimentary information from different modalities images of the same investigated object, which is more suitable for human vision and has advantage on precise clinical diagnosis or further image processing task. We propose a variational image fusion model with adaptively learned tight frame. A data driven wavelet tight frame system is learned from both data to represent the main feature of the source images. The coefficients under this system are combined to construct a fusioned image based on some designed criterion. Numerical experiments on bi-modality images fusion showed that the proposed approach obtained high quality results compared to some existing methods.

This is joint work with Chenglong Bao, Jae-Kyu Choi, and Ying Zhang.

Should one use an educated or uneducated basis?

HONGKAI ZHAO

University of California, Irvine

ABSTRACT

A choice of good basis is important for representation/approximation, analysis and interpretation of quantities of interest. A common difficult balance in real applications is between universality and specificity. This is a really application dependent question. We will show examples for which educated basis, i.e., designing problem specific basis or learning basis, are effective, as well as examples for which uneducated basis, i.e., using simple random basis and bless of dimensions, can also be effective.

Digital Gabor filters and MRA-based Wavelet Tight Frames

Yufei Zhao

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ABSTRACT

Gabor frames, especially digital Gabor filters, have long been known as indispensable tools for local time-frequency analysis of discrete signals. With strong orientation selectivity, tensor products discrete (tight) Gabor frames also see their applications in image analysis and restoration. However, the lack of multiscale structures existing in MRA-based wavelet (tight) frames makes discrete Gabor frames less effective on modeling local structures of signals with varying sizes. Indeed, historically speaking, it was the motivation of studying wavelet systems. By applying the unitary extension principle on some most often seen digital Gabor filters (e.g. local discrete Fourier transform and discrete Cosine transform), we are surprised to find out that these digital filter banks generate MRA-based tight wavelet frames in square integrable function space, and the corresponding refinable functions and wavelets can be explicitly given. In other words, the discrete tight frames associated with these digital Gabor filters can be used as the filter banks of MRA wavelet tight frames, which introduces both multi-scale structures and fast cascade implementation of discrete signal decomposition/reconstruction. Discrete tight frames generated by such filters with both wavelet and Gabor structures can see their potential applications in image processing and recovery.

Influence Prediction for Continuous-Time Information Propagation on Networks Using Graph-Based Fokker-Planck Equation

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ABSTRACT

We consider the problem of predicting influence, defined as the expected number of infected nodes, resulted from information propagating from any given set of source nodes on a network. We develop a novel framework that adaptively aggregates the activation states of the network according to the number of active nodes, leading to the construction of a system of differential equations that governs the time evolution of the state probabilities. This system is analogous to the Fokker-Planck equation in continuous space, and the solution readily yields the desired influence. This approach gives rise to a class of novel and scalable algorithms that work effectively for large-scale and dense networks. Numerical results on a variety of synthetic and real-world networks will be presented. This work is based on the paper reported in [1].

References

[1] S.-N. Chow, X. Ye, H. Zha and H. M. Zhou, *Influence Prediction for Continuous-Time Information Propagation on Networks*, arXiv:1512.05417 (2015).